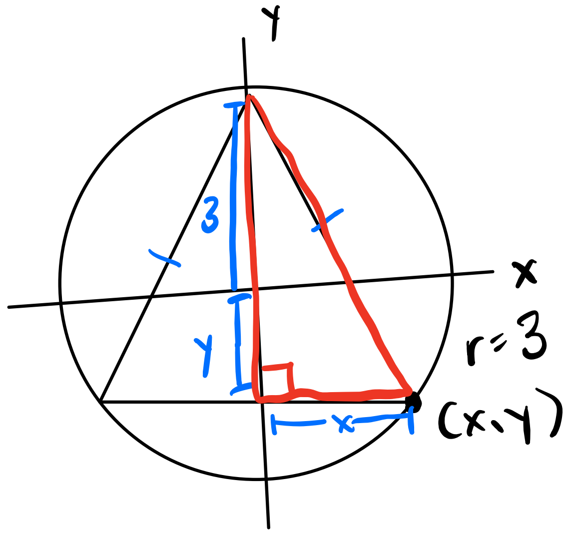
Homework 21

6) largest area of isosceles right triangle inscribe in circle of radius 3



$A = \frac{1}{2}bh$

$b = 2x$

$h = 3 - y$

$A = \frac{1}{2} \cdot 2x(3-y)$

$A = x(3-y)$

$A = \sqrt{9-y^2}\,(3-y)$

$\frac{dA}{dy} = \sqrt{9-y^2}(-1) + (3-y) \cdot \cancel{\frac{1}{2}}(9-y^2)^{-\frac{1}{2}}(-\cancel{2}y)$

$0 = -\sqrt{9-y^2} + \frac{-y(3-y)}{\sqrt{9-y^2}}$

$\left(\frac{\sqrt{9-y^2}}{\sqrt{9-y^2}}\right) \cdot$

$0 = \frac{-(9-y^2) - y(3-y)}{\sqrt{9-y^2}}$

$0 = -9 + y^2 - 3y + y^2$

$0 = 2y^2 - 3y - 9$

$0 = (2y+3)(y-3)$

$y = -\frac{3}{2} \quad y = 3$

$x^2 + y^2 = 9$

$x = \sqrt{9-y^2}$

$\cancel{y=3}$

$x = \sqrt{9-9}$

$\cancel{x=0}$

↑ area

$y = -\frac{3}{2}$

$x = \sqrt{9 - (-\frac{3}{2})^2}$

$x = \sqrt{9 - \frac{9}{4}}$

$x = \sqrt{\frac{36-9}{4}}$

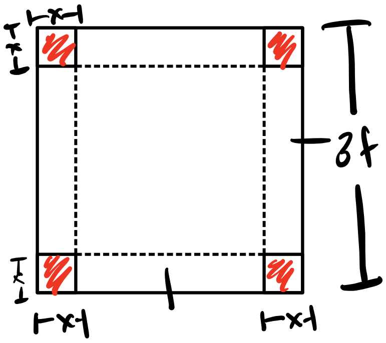
$x = \sqrt{\frac{27}{4}} = \frac{3\sqrt{3}}{2}$

$A = x(3-y)$

$A = \frac{3\sqrt{3}}{2}\left(3 + \frac{3}{2}\right)$

$A = \frac{3\sqrt{3}}{2}\left(\frac{6+3}{2}\right) = \frac{3\sqrt{3}}{2} \cdot \frac{9}{2} = \boxed{\frac{27\sqrt{3}}{4}}$

8)



$V_{max} = ?$

$$V = \overset{l}{(3-2x)}\overset{w}{(3-2x)}\overset{h}{x}$$

$$V = (9 - 12x + 4x^2)x$$

$$V = 4x^3 - 12x^2 + 9x$$

$$\frac{dV}{dx} = 12x^2 - 24x + 9$$

$$0 = 3(4x^2 - 8x + 3)$$

$$0 = 3(2x-3)(2x-1)$$

$$x = \frac{3}{2}, \frac{1}{2}$$

$$x = \cancel{1.5}, 0.5$$

$$\underline{x = \frac{1}{2}}$$

$$V = 4\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)^2 + 9\left(\frac{1}{2}\right)$$

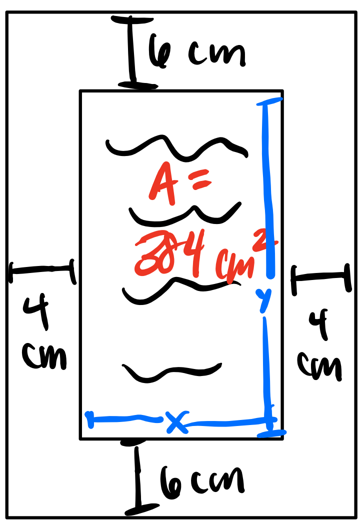
$$V = 4 \cdot \frac{1}{8} - 12 \cdot \frac{1}{4} + \frac{9}{2}$$

$$V = \frac{1}{2} - 3 + \frac{9}{2}$$

$$V = \frac{1}{2} - \frac{6}{2} + \frac{9}{2}$$

$$V = \frac{4}{2} = 2 \leftarrow \text{max}$$

7)



$$A_p = (x + 2 \cdot 4) \cdot (y + 2 \cdot 6) \qquad A_W = xy$$

$$384 = xy$$

$$A_p = (x+8)(y+12) \qquad y = \frac{384}{x}$$

$$A_p = (x+8)\left(\frac{384}{x} + 12\right)$$

$$A_p' = (x+8)(-384x^{-2}) + \left(\frac{384}{x} + 12\right)$$

$$0 = \frac{-384(x+8)}{x^2} + \frac{384}{x} + 12$$

$$0 = \frac{-384(x+8) + 384x + 12x^2}{x^2}$$

$$y = \frac{384}{16} = 24$$

$y_{total} = 24 + 2 \cdot 6$
$= 36$

$x_{total} = 16 + 2 \cdot 4$
$= 24$

$\underline{\underline{24 \times 36}}$

$0 = -\cancel{384}x - 3072 + \cancel{384}x + 12x^2$
$0 = 12x^2 - 3072$
$12x^2 = 3072$
$x^2 = 256$
$x = 16$

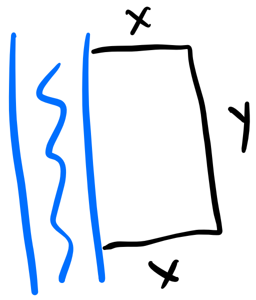
4) positive #s, product = 100, smallest sum

$x, y \in \mathbb{Z}^+$

$xy = 100$
$x = \frac{100}{y}$
$x = \frac{100}{10}$
$\underline{x = 10}$

$s = x + y$
$s = \frac{100}{y} + y$
$s' = -100y^{-2} + 1$
$0 = -100y^{-2} + 1$
$0 = \frac{-100}{y^2} + 1$
$\frac{100}{y^2} = 1$
$y^2 = 100$
$\underline{y = 10}$

1) 2400 ft fence   Amax



$P = x + y + x$

$P = 2x + y$

$2400 = 2x + y$

$y = 2400 - 2x$

$y = 2400 - 2 \cdot 600$

$y = 2400 - 1200$

$\underline{y = 1200}$

$A = xy$

$A = x(2400 - 2x)$

$A = 2400x - 2x^2$

$A' = 2400 - 4x$

$0 = 2400 - 4x$

$4x = 2400$

$\underline{x = 600}$

Homework 19 (??)

13) $\lim_{x \to 1} (2-x)^{\tan(\frac{\pi}{2}x)}$

$= \lim_{x \to 1} e^{\ln(2-x)^{\tan(\frac{\pi}{2}x)}}$

$= \lim_{x \to 1} e^{\tan(\frac{\pi}{2}x)\ln(2-x)}$

$= \lim_{x \to 1} e^{\frac{\ln(2-x)}{\cot(\frac{\pi}{2}x)}} = \lim_{x \to 1} e^{\frac{\frac{-1}{2-x}}{-\csc^2(\frac{\pi}{2}x) \cdot \frac{\pi}{2}}} = \lim_{x \to 1} e^{\frac{\frac{-1}{2-x}}{-\frac{\pi}{2}\csc^2(\frac{\pi}{2}x)}}$

$= e^{\frac{\frac{-1}{2-1}}{-\frac{\pi}{2}\csc^2(\frac{\pi}{2})}} = e^{\frac{-1}{-\frac{\pi}{2} \cdot 1}} = e^{\frac{1}{\frac{\pi}{2}}} = \boxed{e^{\frac{2}{\pi}}}$