Homework 18

5) $f(x) = x^2 \ln x$

$f'(x) = x^2 \cdot \frac{1}{x} + \ln x \cdot 2x$

$= x + 2x\ln x$

$f''(x) = 1 + 2x \cdot \frac{1}{x} + \ln x \cdot 2$

$= 1 + 2 + 2\ln x$

$= 2\ln x + 3$

$\frac{1}{e} + 2 \cdot \frac{1}{e} \cdot -1 \Rightarrow -$

$\ln \frac{1}{e} = \ln e^{-1} = -\ln e = -1$

$0 = x + 2x\ln x$

$0 = x(1 + 2\ln x)$

$x = 0$ or $1 + 2\ln x = 0$

$2\ln x = 1$

$\ln x = -\frac{1}{2}$

$x = e^{-\frac{1}{2}}$

$x = \frac{1}{\sqrt{e}}$

$0 = 2\ln x + 3$

$2\ln x = -3$

$\ln x = -\frac{3}{2}$

$x = e^{-\frac{3}{2}}$

$x = \frac{1}{e^{3/2}}$

$2 \cdot \ln e^{-2} + 3$

$2 \cdot -2 \cdot 1 + 3$

$-4 + 3$

$-$

$f'(x)$

increasing: $(\frac{1}{\sqrt{e}}, \infty)$

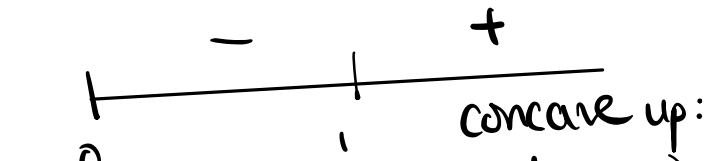
decreasing: $(0, \frac{1}{\sqrt{e}})$

$f(\frac{1}{\sqrt{e}}) = (\frac{1}{\sqrt{e}})^2 \ln(\frac{1}{\sqrt{e}}) = \frac{1}{e} \cdot \ln e^{-\frac{1}{2}}$

$= \frac{1}{e} \cdot -\frac{1}{2} \cdot 1$

$= -\frac{1}{2e}$

$f''(x)$



concave down: $(0, \frac{1}{e^{3/2}})$

inflection point

concave up: $(\frac{1}{e^{3/2}}, \infty)$

$f(\frac{1}{e^{3/2}}) = (\frac{1}{e^{3/2}})^2 \ln e^{-3/2}$

$= \frac{1}{e^3} \cdot -\frac{3}{2} \ln e$

$= -\frac{3}{2e^3}$

Homework 19

6) $\lim_{x\to\infty} x\sin\left(\frac{\pi}{x}\right) = \lim_{x\to\infty} \frac{\sin\left(\frac{\pi}{x}\right)}{\frac{1}{x} = x^{-1}} = \lim_{x\to\infty} \frac{\cos\left(\frac{\pi}{x}\right)\cdot \pi \cdot -1x^{-2}}{-x^{-2}}$

$= \lim_{x\to\infty} \pi\cos\left(\frac{\pi}{x}\right) = \pi\cdot 1 = \pi$

9) $\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{e^x-1}\right) = \lim_{x\to 0^+} \frac{e^x-1-x}{x(e^x-1)} = \lim_{x\to 0^+} \frac{e^x-1}{x(e^x)+(e^x-1)}$

$= \lim_{x\to 0^+} \frac{e^x-1}{xe^x+e^x-1} = \lim_{x\to 0^+} \frac{e^x}{xe^x+e^x+e^x} = \lim_{x\to 0^+} \frac{e^x}{e^x(x+2)} = \lim_{x\to 0^+} \frac{1}{x+2}$

$= \frac{1}{2}$