

Homework 15

$$7) f(x) = \sqrt{x+3} \quad @ a=22$$

$$L(x) = f(a) + f'(a)(x-a)$$

$$f(22) = \sqrt{22+3} = \sqrt{25} = 5$$

$$f'(x) = \frac{1}{2}(x+3)^{-\frac{1}{2}}$$

$$f'(22) = \frac{1}{2}(22+3)^{-\frac{1}{2}}$$

$$= \frac{1}{2}(25)^{-\frac{1}{2}}$$

$$= \frac{1}{2} \cdot \frac{1}{5}$$

$$= \frac{1}{10}$$

$$L(x) = 5 + \frac{1}{10}(x-22)$$

$$\sqrt{24.9}$$

$$\sqrt{25.01}$$

$$\sqrt{x+3} = \sqrt{24.9}$$

$$x+3 = 24.9$$

$$x = 21.9$$

$$L(21.9) = 5 + \frac{1}{10}(21.9-22)$$

$$= 5 + \frac{1}{10}(-0.1)$$

$$= 5 + -0.01$$

$$= 4.99$$

$$\sqrt{25.01} = \sqrt{x+3}$$

$$25.01 = x+3$$

$$x = 22.01$$

$$L(22.01) = 5 + \frac{1}{10}(22.01-22)$$

$$= 5 + \frac{1}{10}(0.01)$$

$$= 5 + 0.001$$

$$= 5.001$$

$$9) e^{0.2} \quad f(x) = e^x \quad @ \quad a=0$$

$$L(x) = f(a) + f'(a)(x-a) \quad e^{0.2}$$

$$f(0) = e^0 = 1$$

$$f'(x) = e^x$$

$$f'(0) = e^0 = 1$$

$$L(0.2) = 1 + 0.2 \\ = 1.2$$

$$L(x) = 1 + 1(x-0) \\ = 1 + x$$

Homework 16

$$5) f(x) = x^2 \sqrt{1-x^2}$$

$$f'(x) = x^2 \cdot \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x) + \sqrt{1-x^2} \cdot 2x$$

$$0 = \frac{-x^3}{\sqrt{1-x^2}} + 2x\sqrt{1-x^2} \left(\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}} \right)$$

$$0 = \frac{-x^3}{\sqrt{1-x^2}} + \frac{2x(1-x^2)}{\sqrt{1-x^2}}$$

$$0 = \frac{-x^3 + 2x - 2x^3}{\sqrt{1-x^2}}$$

$$-3x^3 + 2x = 0$$

$$x(-3x^2 + 2) = 0$$

$$\underline{x=0} \quad \text{or} \quad -3x^2 + 2 = 0$$

$$3x^2 = 2$$

$$x^2 = \frac{2}{3}$$

$$\underline{x = \pm \sqrt{\frac{2}{3}}}$$

$$\sqrt{1-x^2} = 0$$

$$1-x^2 = 0$$

$$x^2 = 1$$

$$\underline{x = \pm 1}$$

$$8) f(x) = \frac{\ln x}{x} \quad [1, 3]$$

$$f'(x) = \frac{x \cdot \frac{1}{x} - \ln x}{x^2} = \frac{1 - \ln x}{x^2}$$

$$0 = \frac{1 - \ln x}{x^2}$$

$$0 = 1 - \ln x$$

$$\ln x = 1$$

$$x = e$$

$$f(1) = \frac{\ln 1}{1} = 0 \quad \leftarrow \text{abs min}$$

$$f(e) = \frac{\ln e}{e} = \frac{1}{e} \quad \leftarrow \text{abs max}$$

$$f(3) = \frac{\ln 3}{3}$$

$$15) f(x) = \begin{cases} 6(x-3)^3 - 1, & x < 4 \\ \ln(x-3) - 10 & x \geq 4 \end{cases} \quad [4, 14]$$

continuous NO

closed, bounded ✓

$$\begin{aligned} \lim_{x \rightarrow 4^-} f(x) &= 6(4-3)^3 - 1 \\ &= 6 \cdot 1 - 1 \\ &= 5 \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow 4^+} f(x) &= \ln(4-3) - 10 \\ &= \ln 1 - 10 \\ &= 0 - 10 \\ &= -10 \end{aligned}$$

EVT: NO