# Homework 18

4) $f(x) = \cos^2 x - 2\sin x \quad [0, 2\pi]$

$f'(x) = 2\cos x \cdot -\sin x - 2\cos x$

$= -2\sin x\cos x - 2\cos x$

$f''(x) = -2\sin x \cdot -\sin x + \cos x \cdot -2\cos x + 2\sin x$

$= 2\sin^2 x - 2\cos^2 x + 2\sin x$

$f'(x)$

$-2\sin x\cos x - 2\cos x = 0$

$-2\cos x(\sin x + 1) = 0$

$-2\cos x = 0$

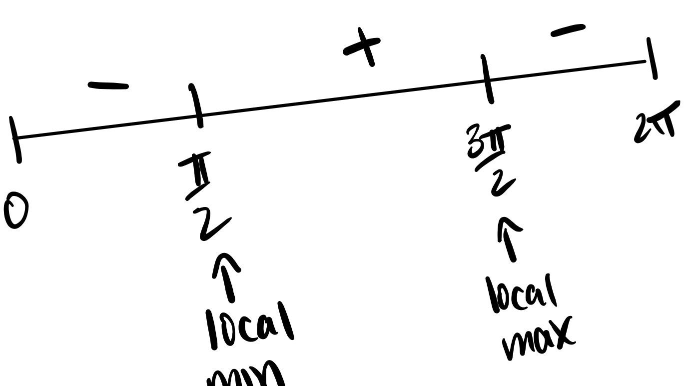
$\cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin x + 1 = 0$

$\sin x = -1$

$x = \frac{3\pi}{2}$




$f(\frac{\pi}{2}) = \cos^2(\frac{\pi}{2}) - 2\sin(\frac{\pi}{2})$

$= 0 - 2 \cdot 1 = -2$

$f(\frac{3\pi}{2}) = \cos^2(\frac{3\pi}{2}) - 2\sin(\frac{3\pi}{2})$

$= 0 - 2 \cdot -1$

$= 2$

$f''(x)$

$2\sin^2 x - 2\cos^2 x + 2\sin x = 0$

$2\sin^2 x - 2(1 - \sin^2 x) + 2\sin x = 0$

$2\sin^2 x - 2 + 2\sin^2 x + 2\sin x = 0$

$4\sin^2 x + 2\sin x - 2 = 0$

$2(2\sin^2 x + \sin x - 1) = 0$

$u = \sin x$

$2(2u^2 + u - 1) = 0$

$2(2u - 1)(u + 1) = 0$

$2(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0$

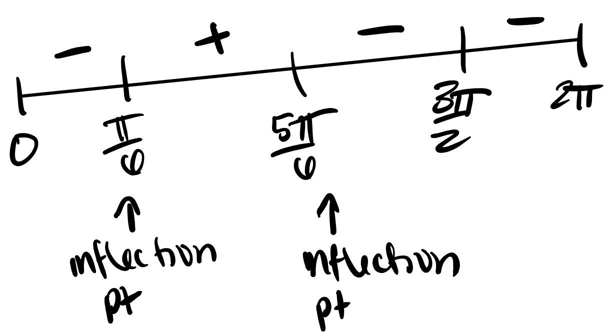
$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\sin x + 1 = 0$

$\sin x = -1$

$x = \frac{3\pi}{2}$



increasing : $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ $\qquad f\left(\frac{\pi}{6}\right) = -\frac{1}{4}$

decreasing : $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right]$ $\qquad f\left(\frac{5\pi}{6}\right) = -\frac{1}{4}$

Homework 19

6) $\lim_{x \to \infty} x\sin\left(\frac{\pi}{x}\right) = \lim_{x \to \infty} \frac{\sin\left(\frac{\pi}{x}\right)}{\frac{1}{x}} = \lim_{x \to \infty} \frac{\cos\left(\frac{\pi}{x}\right) \cdot \pi \cdot \cancel{-1x^{-2}}}{\cancel{-1x^{-2}}}$

$= \lim_{x \to \infty} \cos\left(\frac{\pi}{x}\right) \cdot \pi = \cos(0) \cdot \pi = 1 \cdot \pi = \pi$

9) $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{e^x - 1}\right) = \lim_{x \to 0^+} \frac{e^x - 1 - x}{x(e^x - 1)} = \lim_{x \to 0^+} \frac{e^x - 1}{xe^x + (e^x - 1)}$

$= \lim_{x \to 0^+} \frac{e^x - 1}{xe^x + e^x - 1} = \lim_{x \to 0^+} \frac{e^x}{xe^x + e^x + e^x} = \lim_{x \to 0^+} \frac{e^x}{e^x(x+2)}$

$= \lim_{x \to 0^+} \frac{1}{x+2} = \frac{1}{2}$