FR4

$A = \frac{1}{2}bh$

$A = \frac{1}{2} \cdot 2xh$

$A = xh$

$x^2 + h^2 = 6^2$

$h = \sqrt{36 - x^2}$

$A = x\sqrt{36 - x^2}$

$A' = x \cdot \frac{1}{\cancel{2}}(36 - x^2)^{-\frac{1}{2}}(-\cancel{2}x) + \sqrt{36 - x^2}$

$0 = \frac{-x^2}{\sqrt{36 - x^2}} + \sqrt{36 - x^2}\left(\frac{\sqrt{36 - x^2}}{\sqrt{36 - x^2}}\right)$

$0 = \frac{-x^2 + 36 - x^2}{\sqrt{36 - x^2}}$

$0 = -2x^2 + 36$

$2x^2 = 36$

$x^2 = 18$

$x = 3\sqrt{2}$

$A = 3\sqrt{2}\sqrt{36 - (3\sqrt{2})^2}$

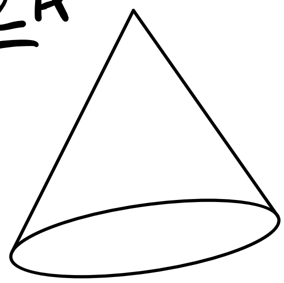
$A = 3\sqrt{2}\sqrt{36 - 18}$

$A = 3\sqrt{2} \cdot \sqrt{18}$

$A = 3\sqrt{2} \cdot 3\sqrt{2}$

$A = 18$

FR2A



$r = 2h$

$\frac{dV}{dt} = 12\pi$

$\frac{dh}{dt} = ?$

$r = 2$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (2h)^2 h$

$V = \frac{1}{3}\pi \cdot 4h^2 h$

$V = \frac{\pi}{3} \cdot 4h^3$

$\frac{dV}{dt} = 4\pi h^2 \frac{dh}{dt}$

$2 = 2h$

$h = 1$

$12\pi = 4\pi \cdot 1^2 \frac{dh}{dt}$

$3 = 1 \frac{dh}{dt}$

$\frac{dh}{dt} = 3$

B

$r = 2h$

$\frac{dV}{dt} = 54\pi$

$\frac{dh}{dt} = ?$

$r = 3$

$3 = 2h$

$h = \frac{3}{2}$

$\frac{dV}{dt} = 4\pi h^2 \frac{dh}{dt}$

$54\pi = 4\pi \left(\frac{3}{2}\right)^2 \frac{dh}{dt}$

$54\pi = \cancel{4}\pi \cdot \frac{9}{\cancel{4}} \frac{dh}{dt}$

$54\pi = 9\pi \frac{dh}{dt}$

$\frac{dh}{dt} = 6$

FR3

$g(x) = ax^2 + x + k \quad [0,1]$

$\frac{f(b) - f(a)}{b - a} = f'(c)$

$f(1) = a + 1 + k$

$f(0) = k$

$f'(x) = 2ax + 1$

$\frac{a + 1 + \cancel{k} - \cancel{k}}{1 - 0} = 2ac + 1$

$a + 1 = 2ac + 1$

$a = 2ac$

$c = \frac{1}{2}$

Homework 27

$u = \ln x \quad du = \frac{1}{x}dx$

14) $\int_1^e \frac{8(\ln x)^3}{x}dx = \int 8u^3 du = 2u^4 \Big| = 2(\ln x)^4 \Big|_1^e$

$= 2(\ln e)^4 - 2(\ln 1)^4 = 2 \cdot 1 - 2 \cdot 0 = 2$

$dx = \frac{du}{-3}$

$u = -3x \quad du = -3dx$

6) $\int_{-1}^{1} 12e^{-3x}dx = -4\int_{-1}^{1} -3e^{-3x}dx = -4\int e^{u}du$

$= -4e^{-3x}\Big|_{-1}^{1} = -4e^{-3} + 4e^{3}$

Lecture 25

12) $h(x) = \int_{2x}^{x^2} \cos(t)\sin(t)dt$

$h'(x) = \cos(x^2)\sin(x^2) \cdot 2x - \cos(2x)\sin(2x) \cdot 2$

10) $h(x) = \int_0^x t\sin(t)dt \qquad h'(t) = x\sin(x)$

11) $h(x) = \int_x^2 t + e^t dt \qquad h'(t) = -(x + e^x)$

27:15

$$\int_1^2 \frac{8x^3+4}{\sqrt{x^4+2x}}\,dx \qquad u = x^4+2x \quad du = 4x^3+2\,dx$$

$$= 2\int_1^2 \frac{4x^3+2}{\sqrt{x^4+2x}}\,dx = 2\int u^{-\frac{1}{2}}\,du = 2\cdot 2u^{\frac{1}{2}} \Big|$$

$\frac{du}{u^{\frac{1}{2}}}$

$$= 4\left(x^4+2x\right)^{\frac{1}{2}} \Big|_1^2 = 4(16+4)^{\frac{1}{2}} - 4(1+2)^{\frac{1}{2}}$$

$$= 4\sqrt{20} - 4\sqrt{3} = 8\sqrt{5} - 4\sqrt{3}$$

ex. $$\lim_{x\to 1^+} x^{\frac{1}{x-1}} = \lim_{x\to 1^+} e^{\ln\left(x^{\frac{1}{x-1}}\right)} =$$

$$\lim_{x\to 1^+} e^{\frac{1}{x-1}\ln x} = \lim_{x\to 1^+} e^{\frac{\ln x}{x-1}} = \lim_{x\to 1^+} e^{\frac{\frac{1}{x}}{1}}$$

$$= \lim_{x\to 1^+} e^{\frac{1}{x}} = e^1 = e$$