Evasion Paths in Mobile Sensor Networks

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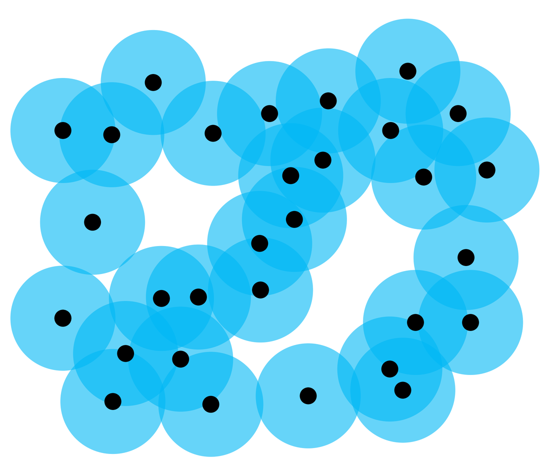
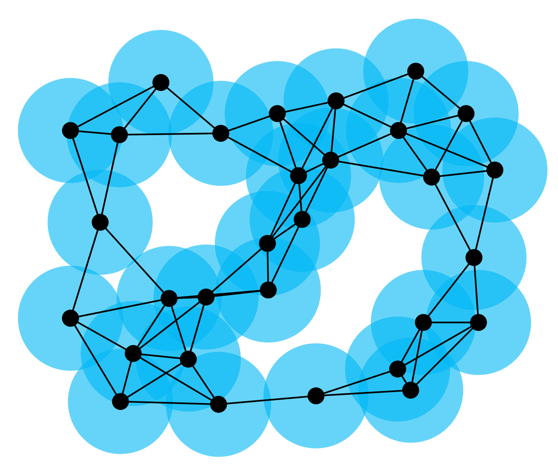
The Evasion Problem

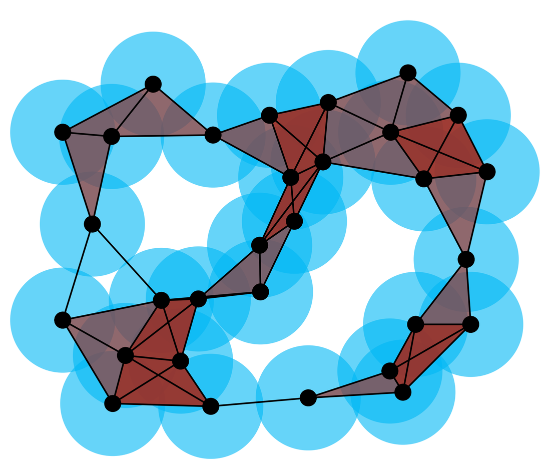
Suppose ball-shaped sensors wander in a bounded and contractible domain $D \subset \mathbb{R}^d$ over time interval $I = [0,1]$. Assume immobile sensors cover the boundary ∂D. The sensors don't know their locations but do measure their time-varying Čech complex (the nerve of the sensor balls). An *evasion path* exists if it is possible for a wandering intruder to avoid the sensors. Using only the time-varying Čech complex, can we determine if an evasion path exists?



Sensor balls



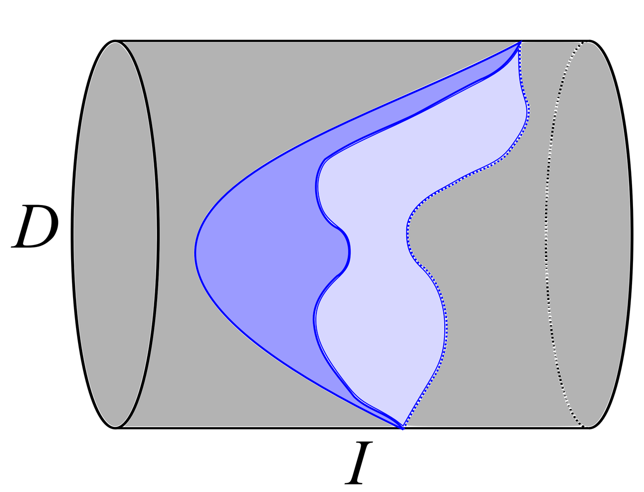
Connectivity graph



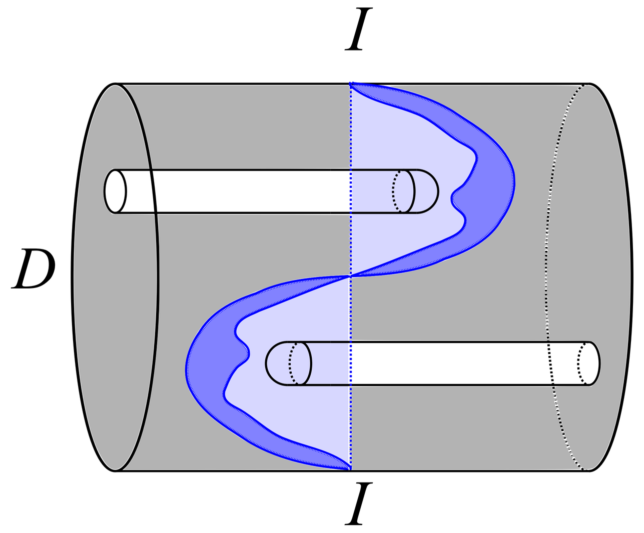
Čech complex

Theorem of de Silva and Ghrist

Theorem [dG06]: *If there is a relative 2-cycle in the region of spacetime covered by sensors with boundary wrapping around $\partial D \times I$, then no evasion path exists.*



A relative 2-cycle.

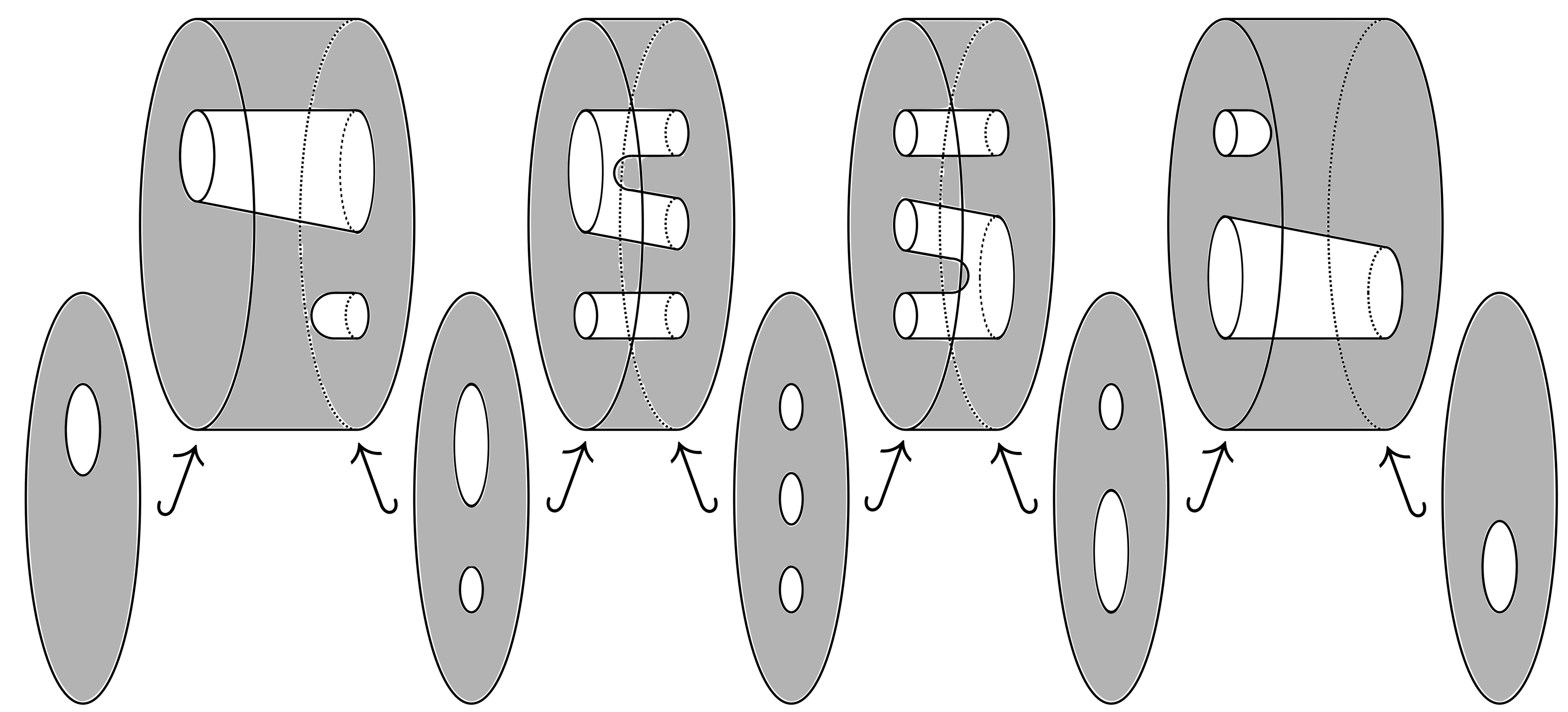


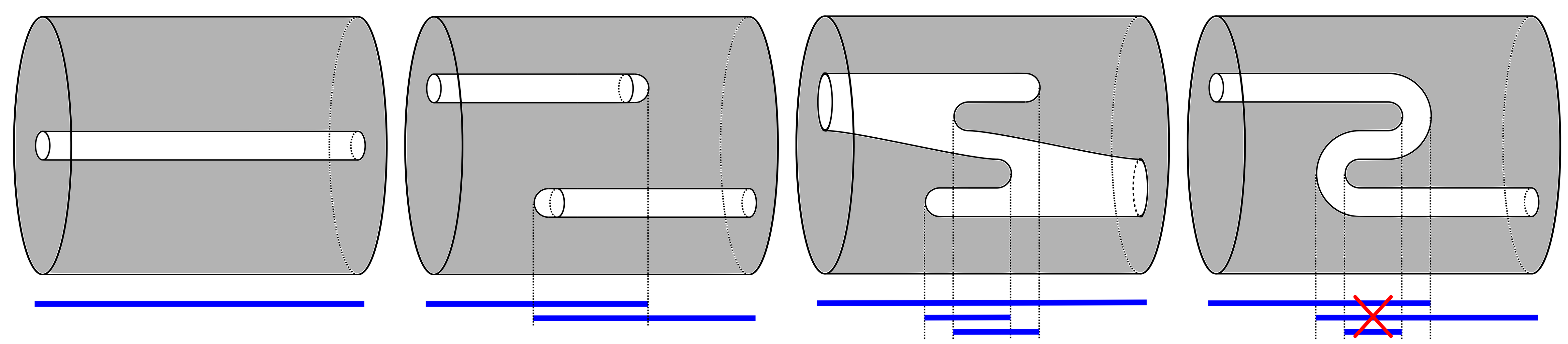
No evasion path exists.

This theorem is not sharp. Can you see why?

Zigzag Persistence

Theorem [Ada13]: *If there is no full-length interval in the $(d-1)$-dimensional zigzag persistent homology of the covered region, then no evasion path exists.*



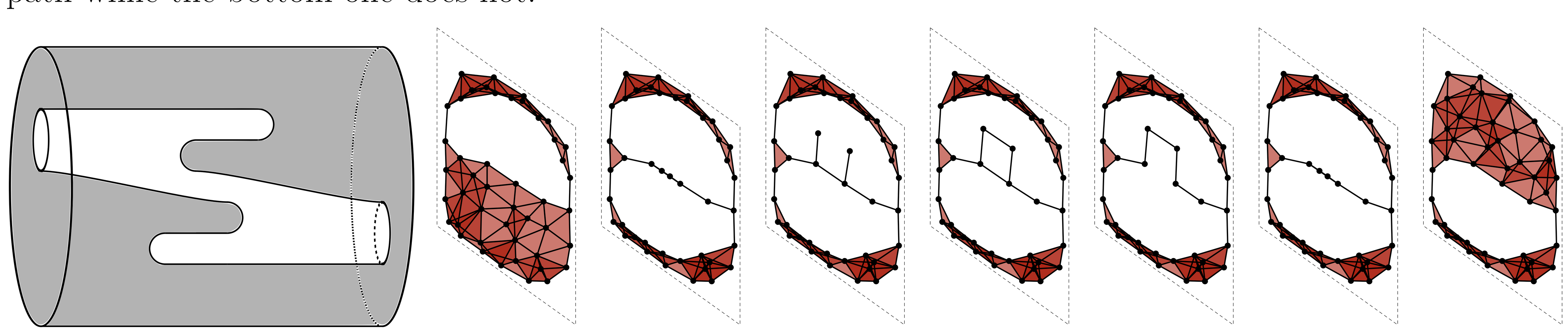


The zigzag persistence barcodes on the right are incorrect. Why?

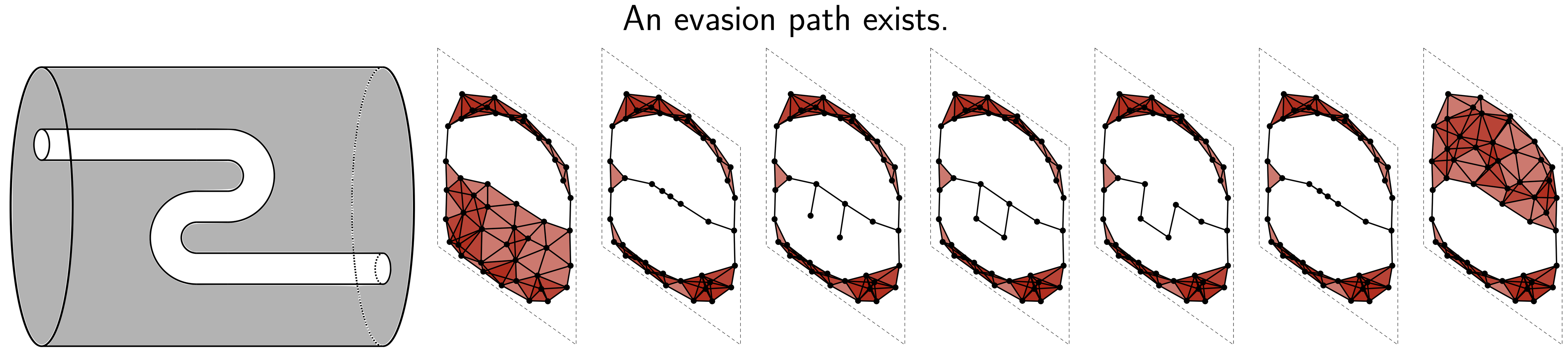
This criterion allows for streaming computation.

Dependence on the Embedding

Neither the time-varying Čech complex nor the fibrewise homotopy type of the covered region determine if an evasion path exists. Consider the two sensor networks below: their time-varying Čech complexes are equal and their covered regions are fibrewise homotopy equivalent, but the top network contains an evasion path while the bottom one does not.



An evasion path exists.



No evasion path exists because the intruder cannot travel backwards in time.

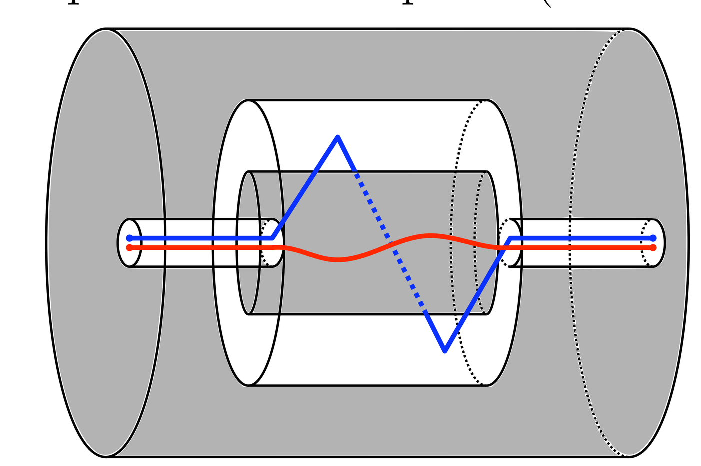
What Minimal Sensing Capabilities Might We Add?

Theorem [Ada13]: *In a planar sensor network that remains connected, the time-varying alpha complex and the cyclic orderings of the edges about each sensor determine if an evasion path exists.*

Open Question: Do the time-varying Čech complex and the cyclic orderings of the edges about each sensor also suffice?

What is the Space of Evasion Paths?

What information do we need about covered region X and its fibrewise embedding in spacetime $D \times I$ to describe the space of evasion paths (sections $I \to X^c$)?



We apply a homotopy spectral sequence for function complexes between diagrams of spaces [DZ87], which in this setting has input depending on unstable invariants of the uncovered region, and which converges to information about the homotopy groups of the space of evasion paths. It remains to obtain these unstable invariants from embedding invariants of the covered region, and one idea is to use the tools of embedding calculus in a fibrewise setting.

References

[Ada13] Henry Adams, *Evasion paths in mobile sensor networks*, PhD Thesis, Stanford University, 2013. arXiv:1308.3536.

[Cd10] Gunnar Carlsson and Vin de Silva, *Zigzag persistence*, Foundations of Computational Mathematics **10** (2010), 367–405.

[dG06] Vin de Silva and Robert Ghrist, *Coordinate-free coverage in sensor networks with controlled boundaries via homology*, International Journal of Robotics Research **25** (2006), 1205–1222.

[DZ87] Emmanuel Dror Farjoun and Alexander Zabrodsky, *The homotopy spectral sequence for equivariant function complexes*, Algebraic Topology Barcelona, Lecture Notes in Mathematics, vol. 1298, Springer, 1987, pp. 54–81.