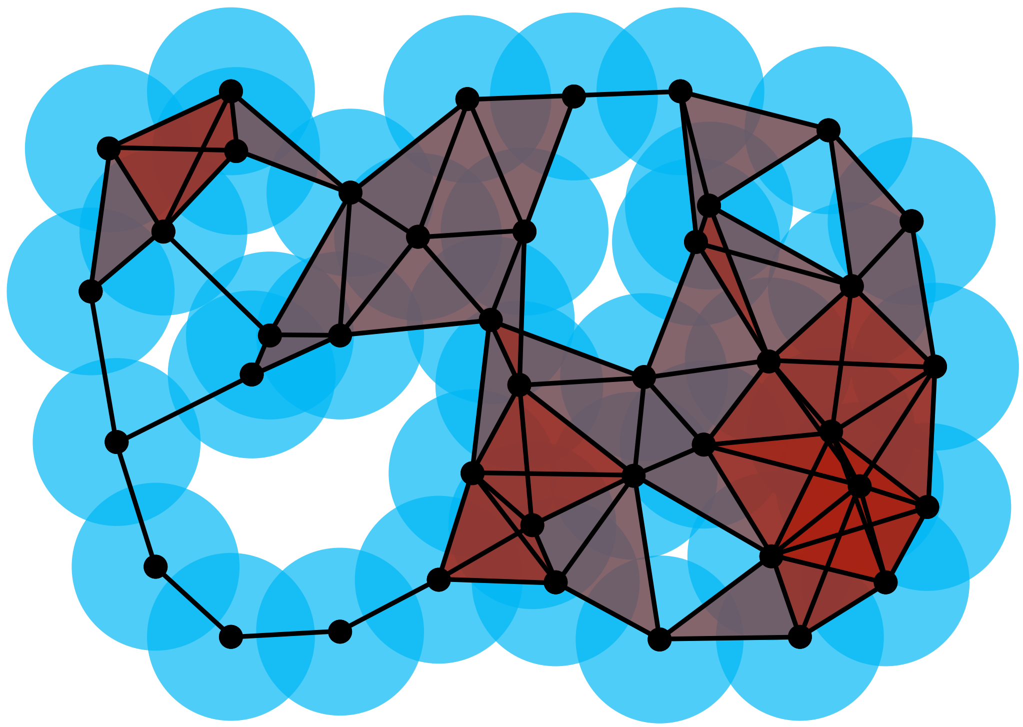
Evasion Paths in Mobile Sensor Networks



Henry Adams, University of Florida

Joint with Gunnar Carlsson

Joint with Deepjyoti Ghosh, Clark Mask, William Ott, Kyle Williams

AATRN

Applied Algebraic Topology Research Network

- www.aatrn.net
- 1-2 live talks per week
- 5,500 YouTube subscribers
- 22 hours watched per day

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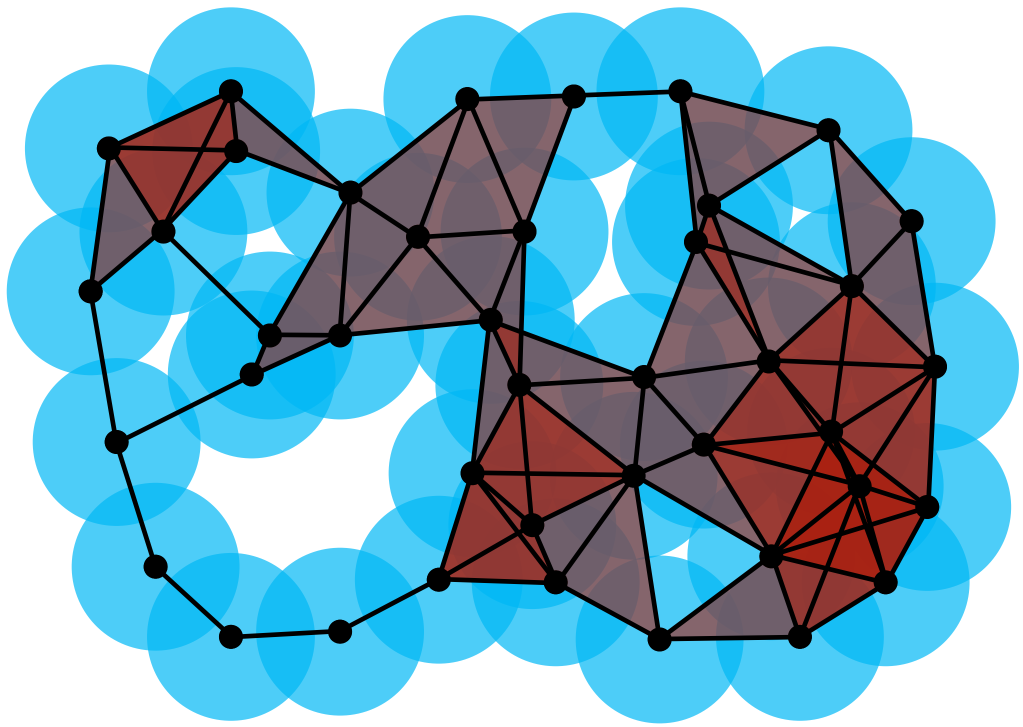

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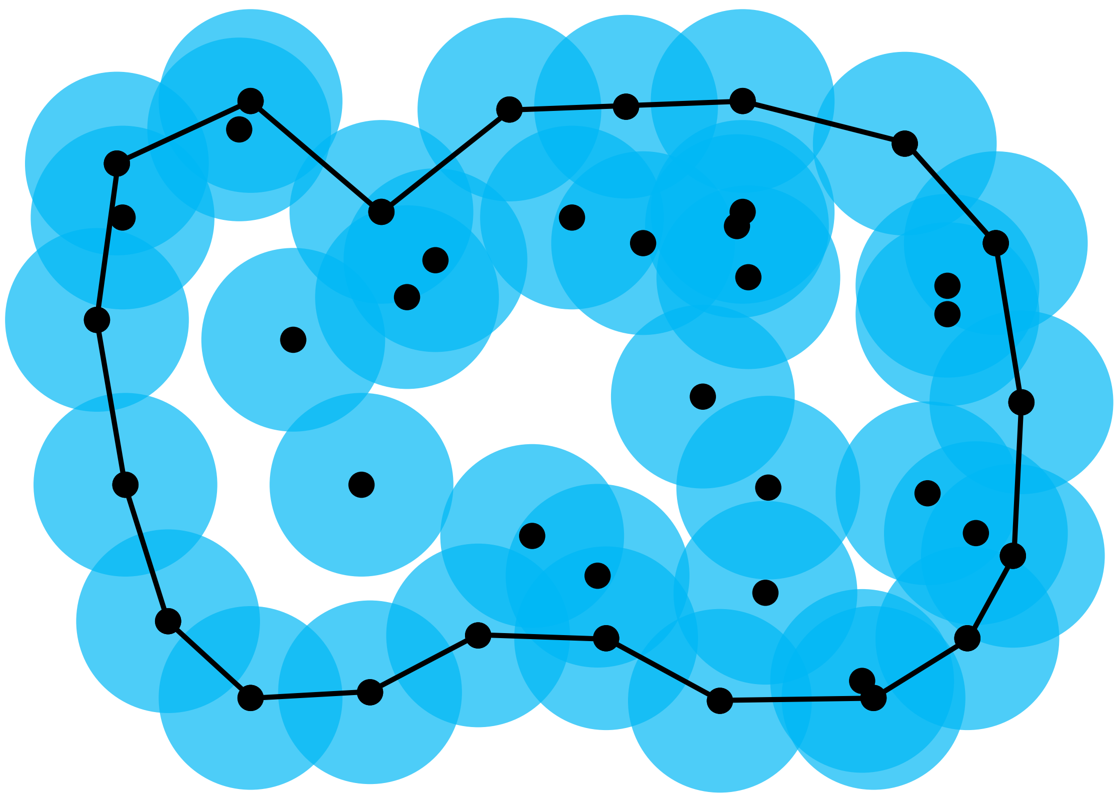
- Sensors move in a ball-shaped domain $B \subset \mathbb{R}^d$ over time interval $I = [0, 1]$. Fixed sensors cover the boundary.
- Measure only the Čech complex.
- Is there an evasion path?



Coordinate-free Coverage in Sensor Networks with Controlled Boundaries via Homology by V. de Silva and R. Ghrist

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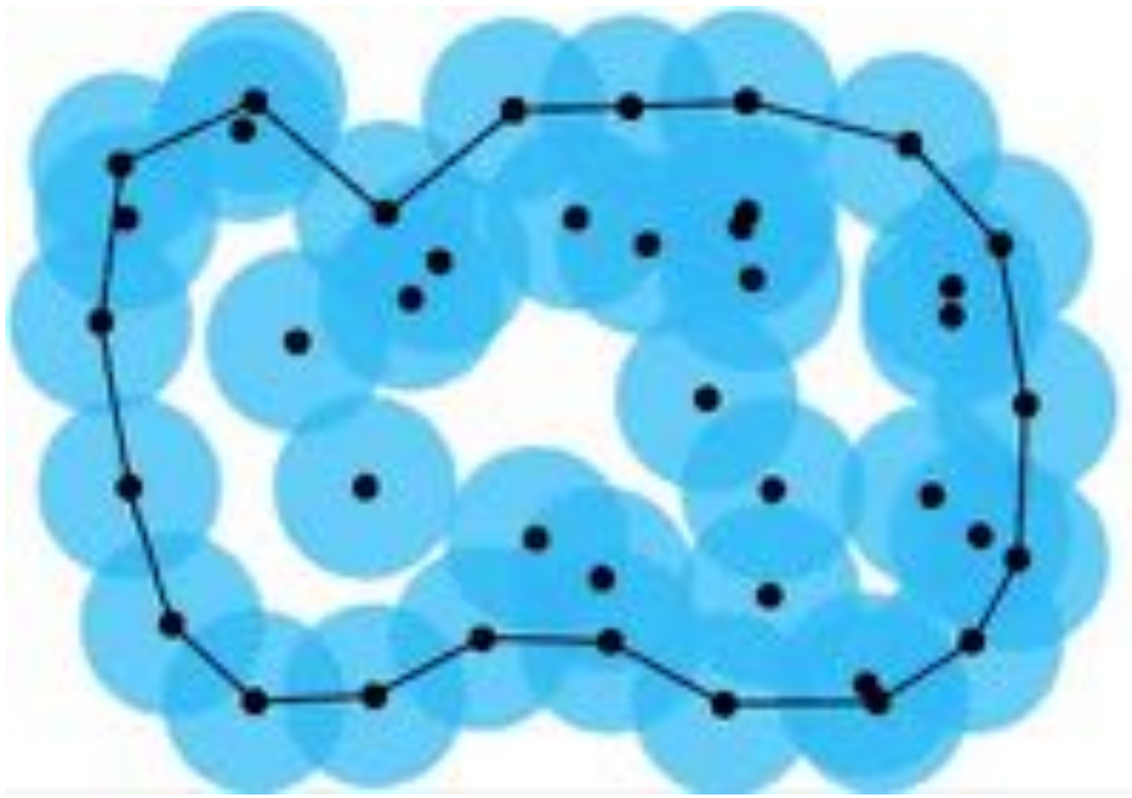

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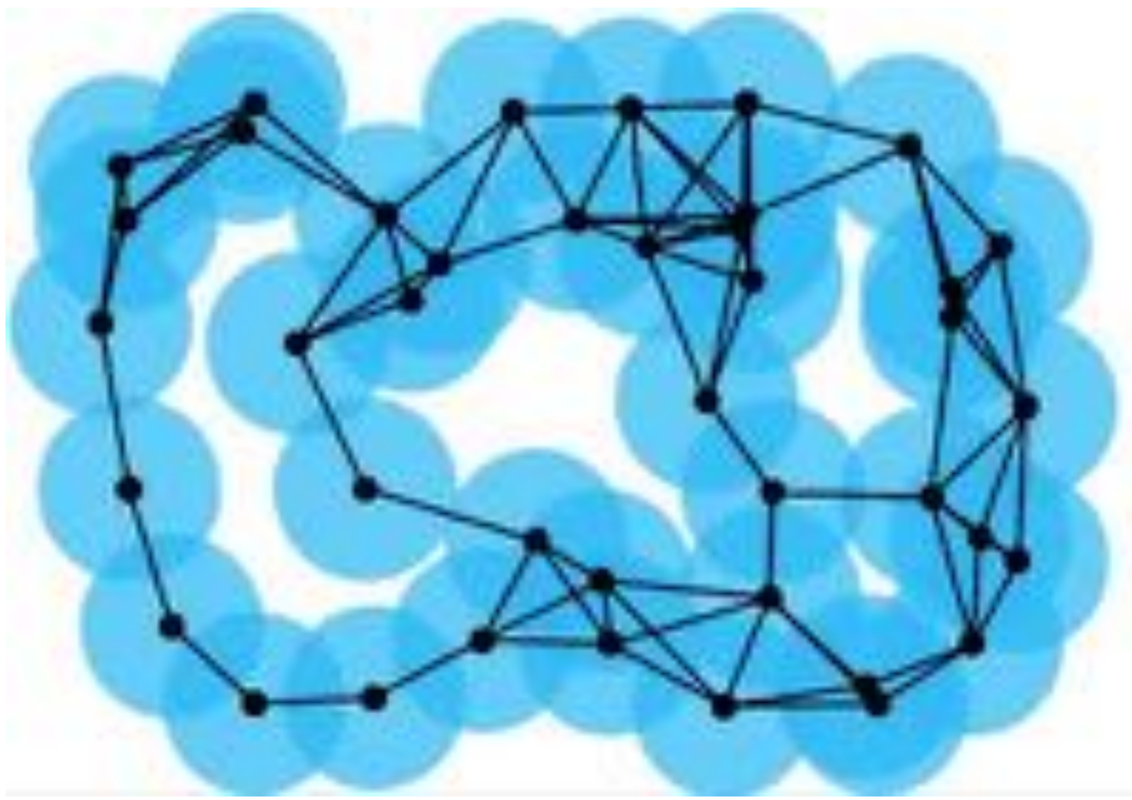

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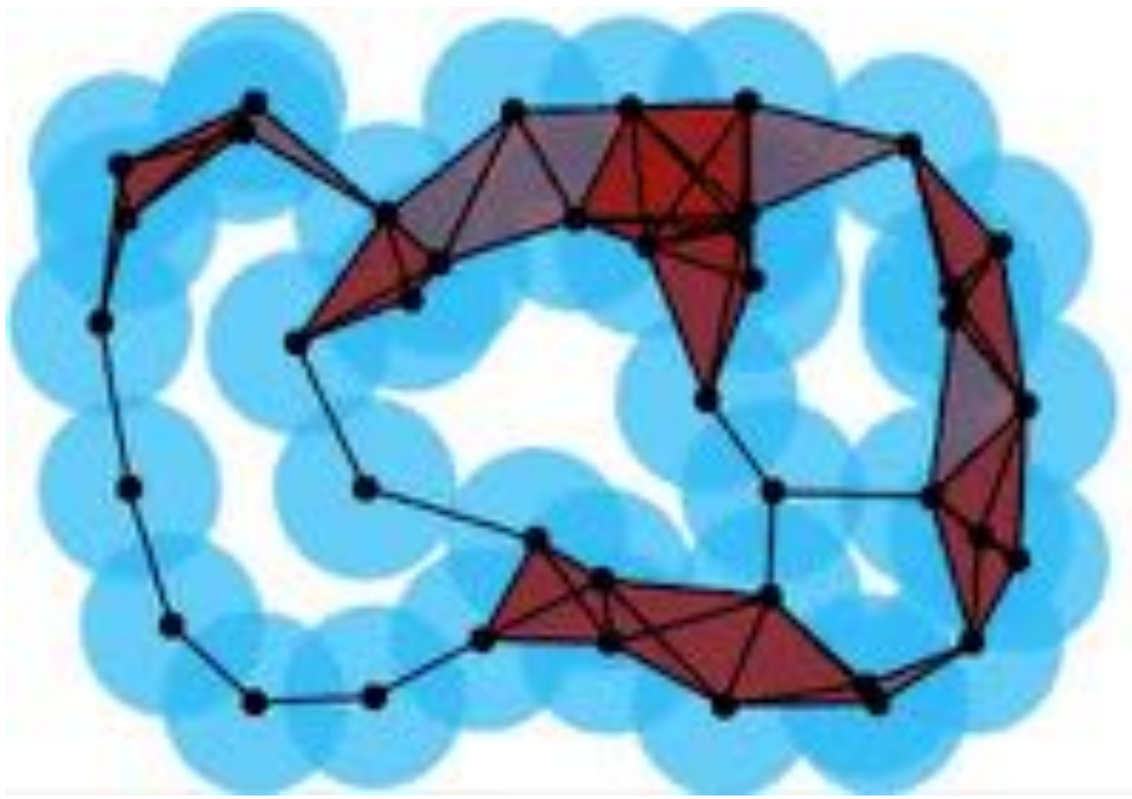

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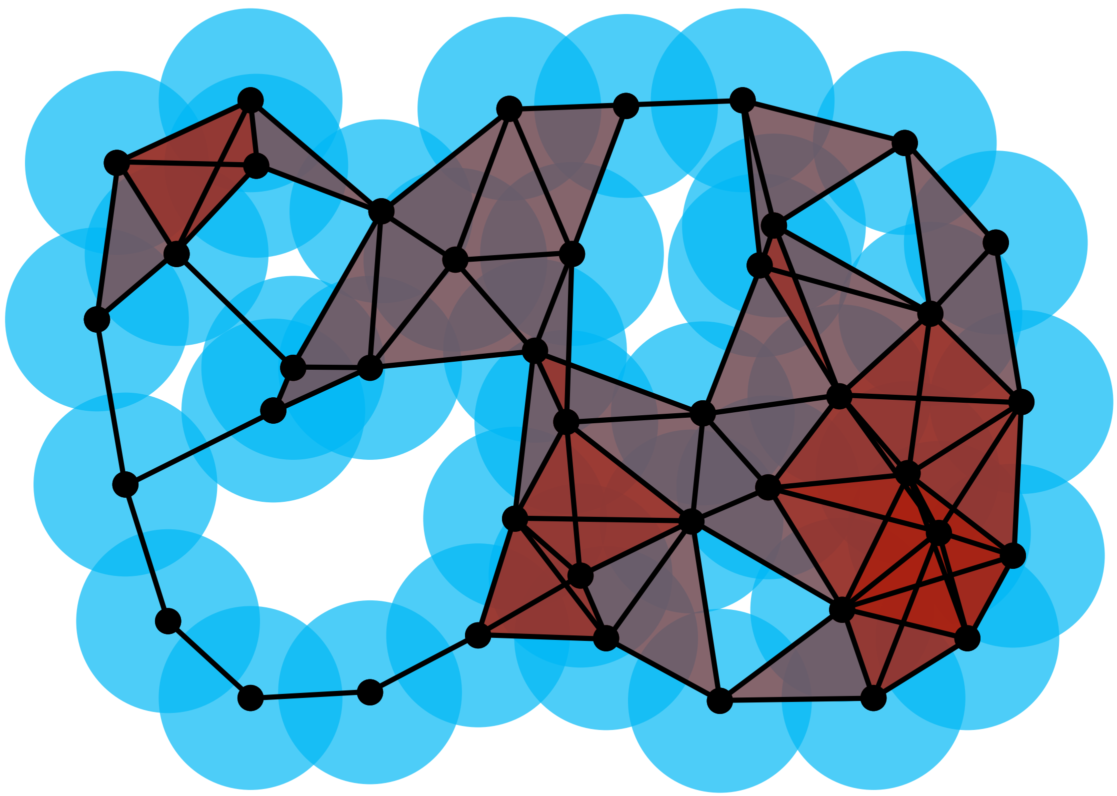

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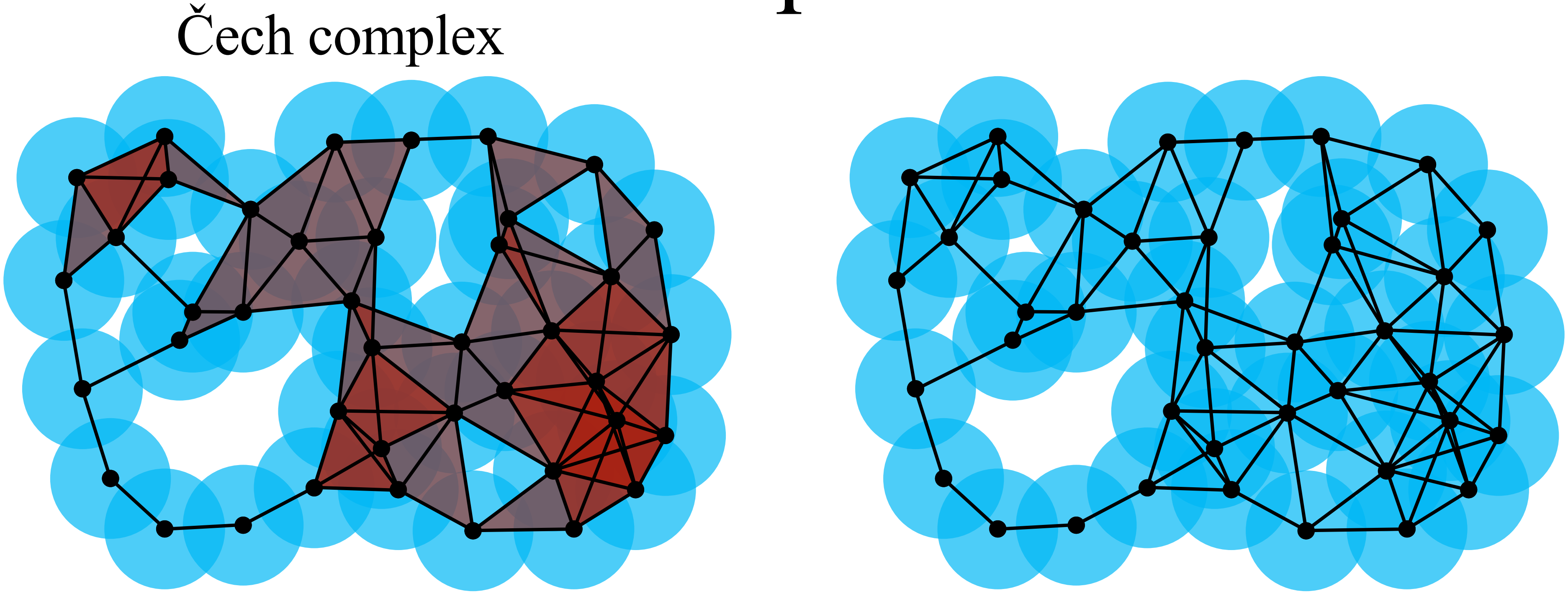

- One vertex for each ball
- Edges when 2 balls overlap
- Triangles when 3 balls overlap

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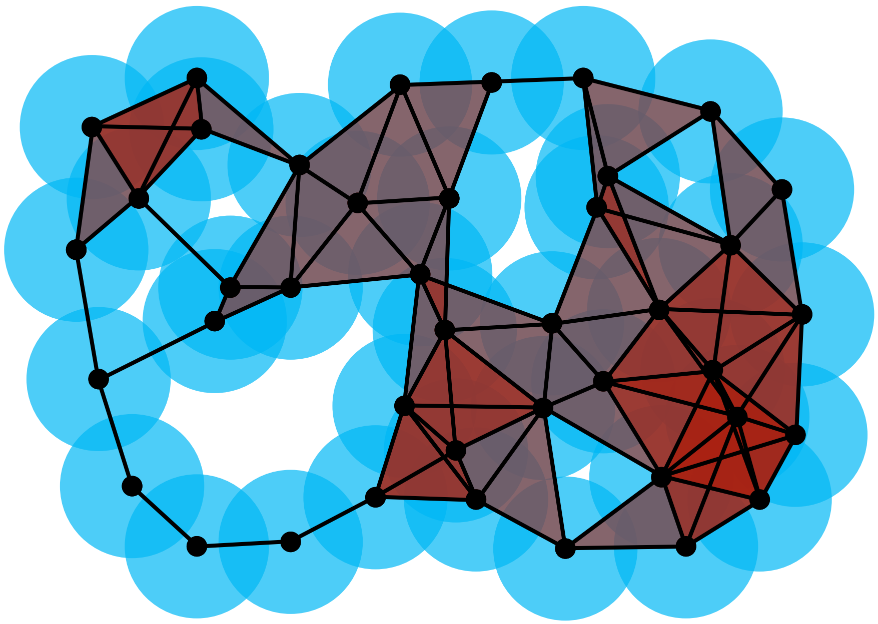
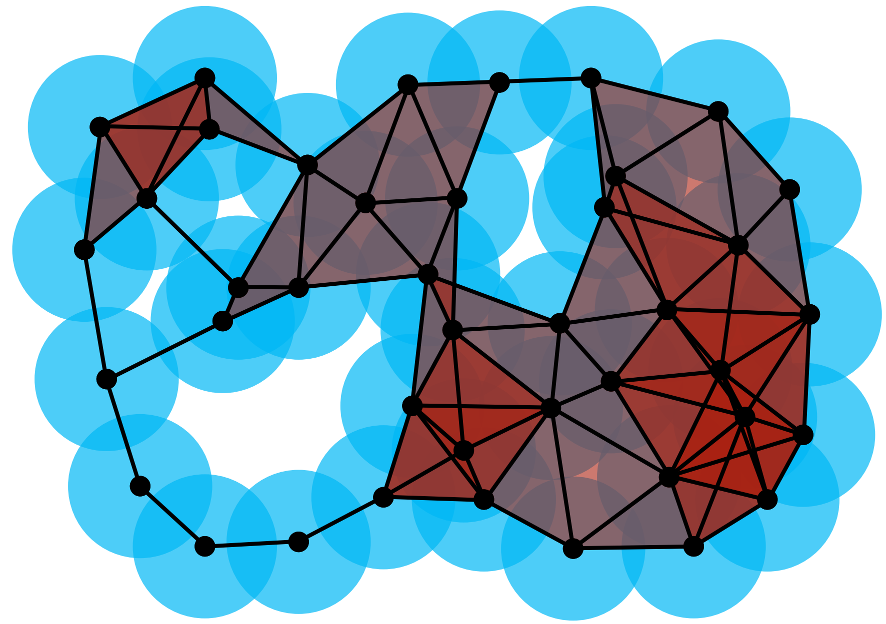
Čech complex



Vietoris-Rips complex



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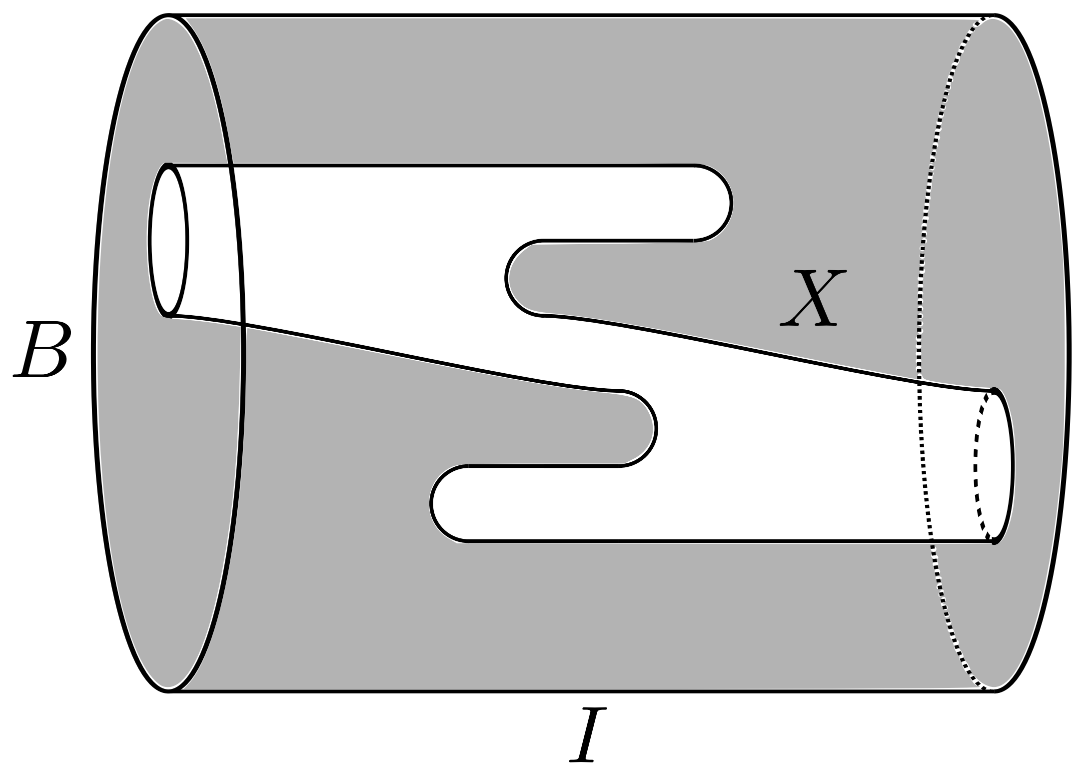

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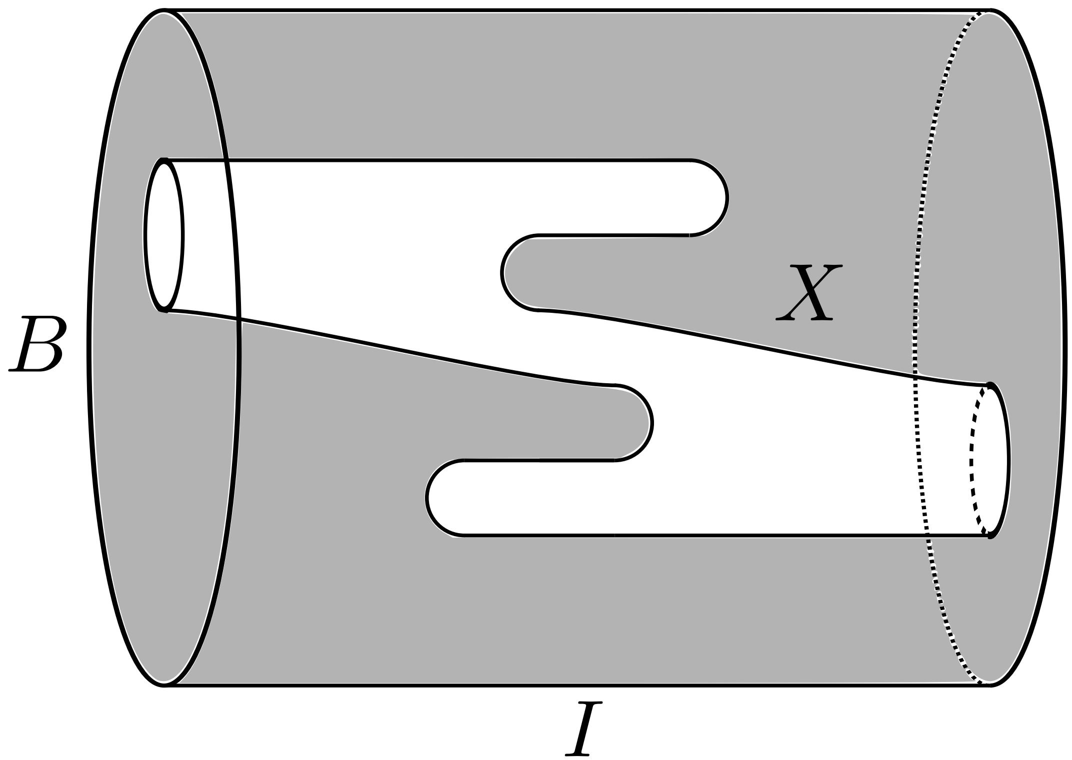

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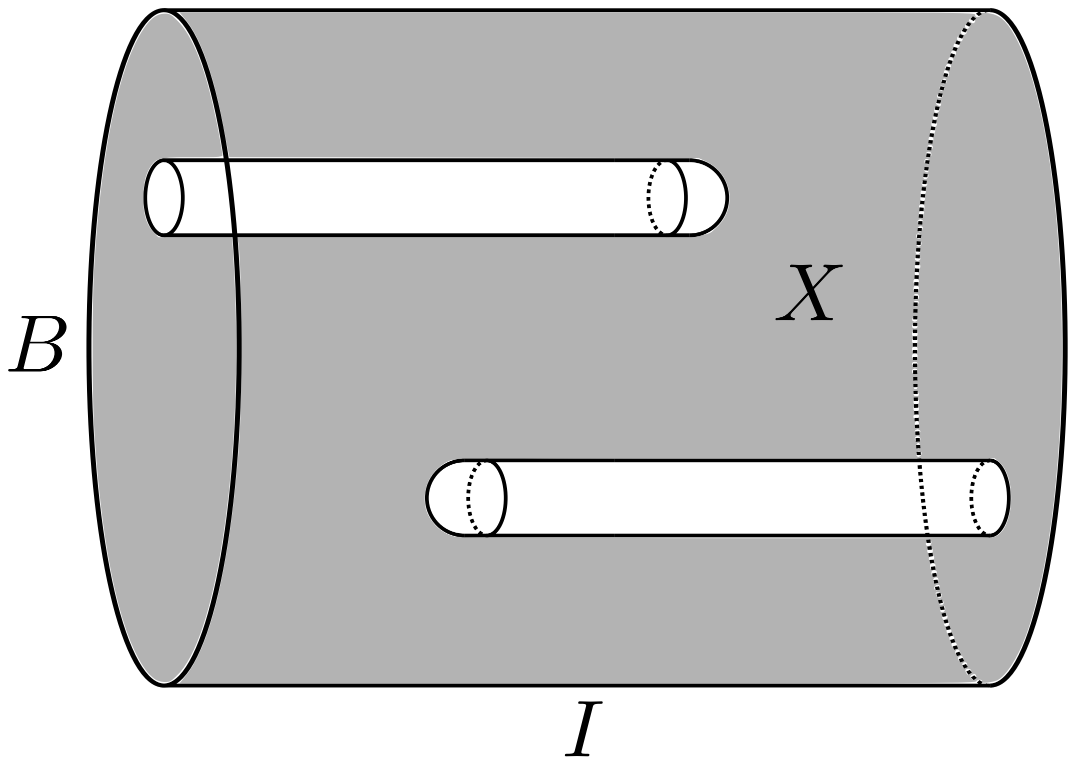

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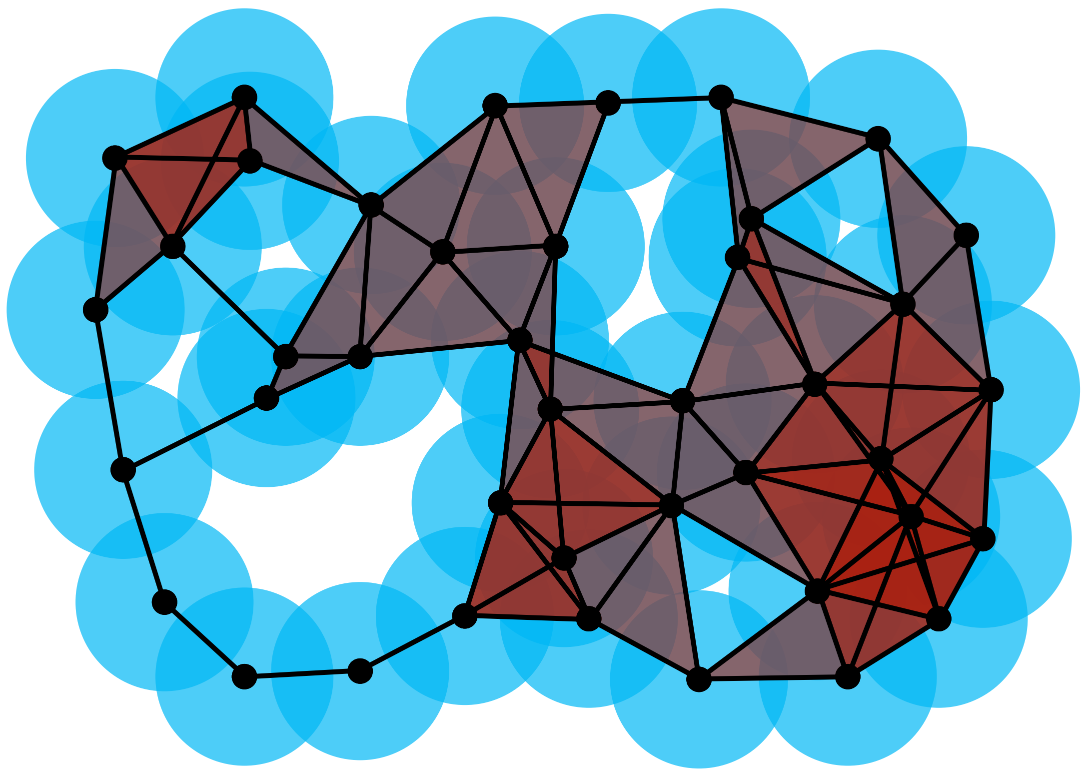

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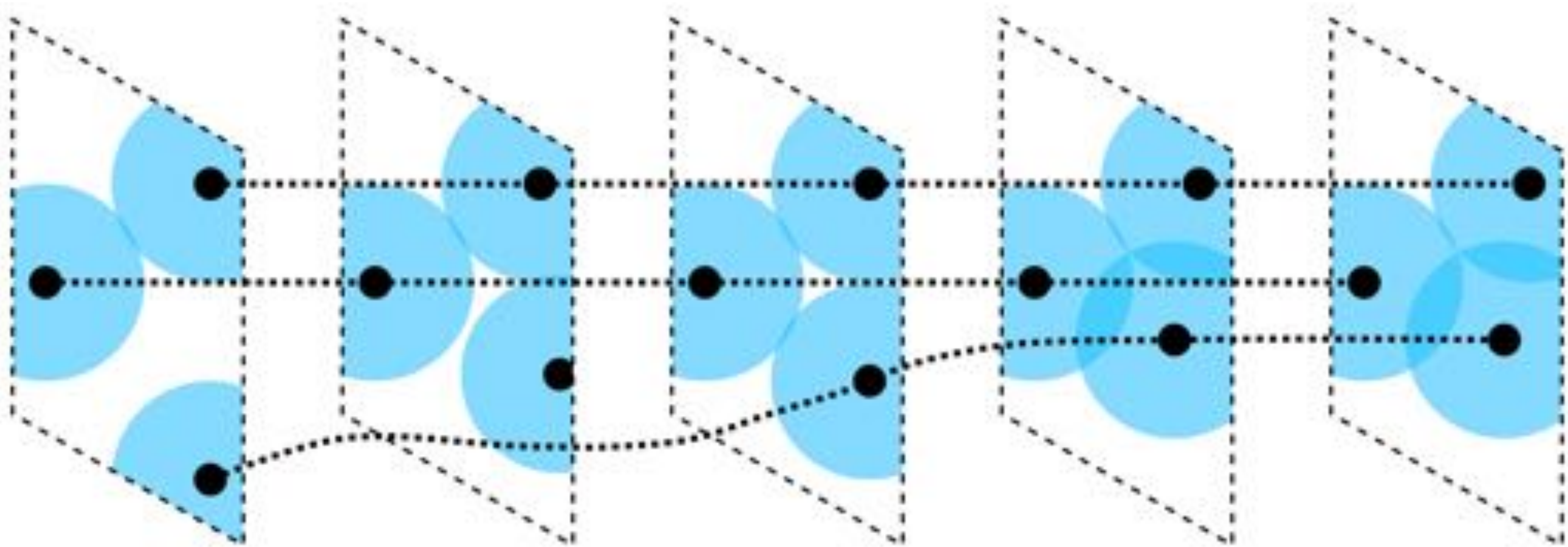

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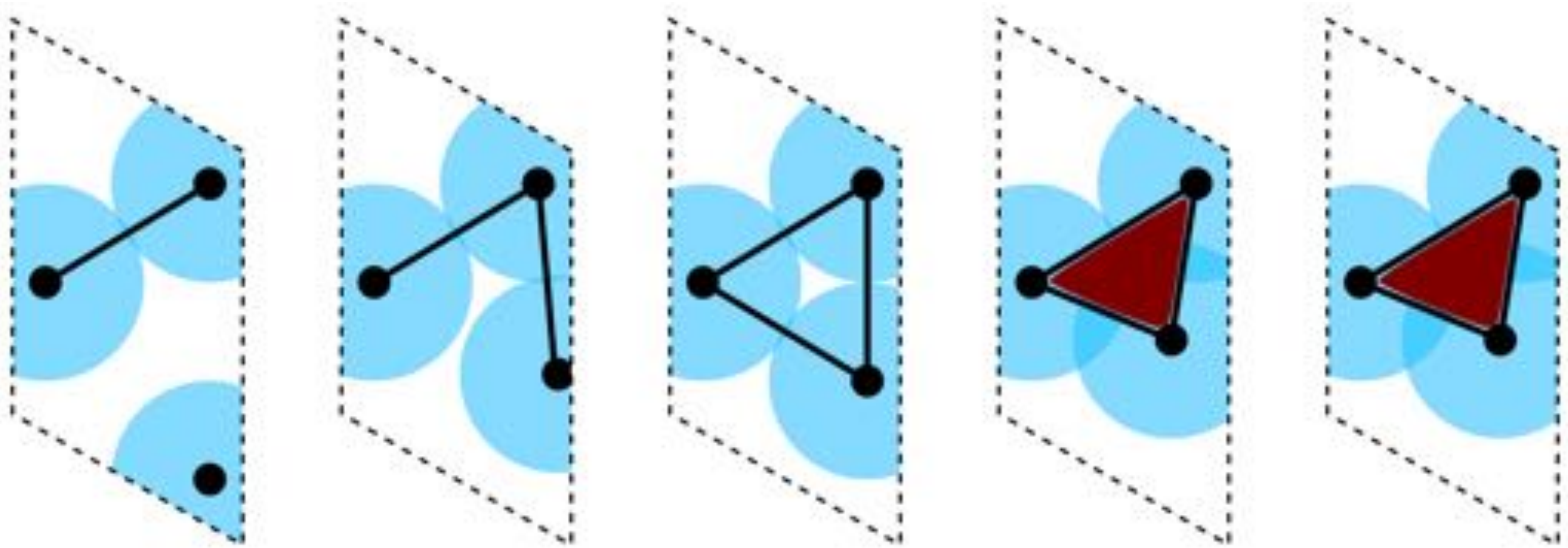

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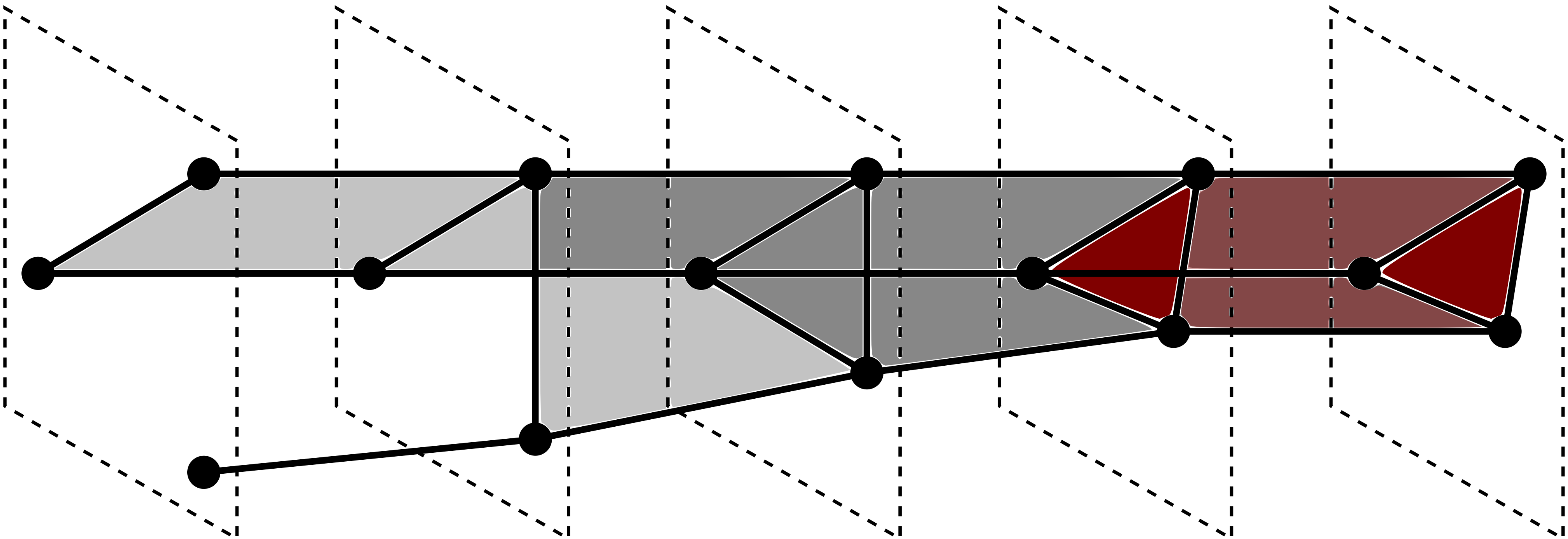

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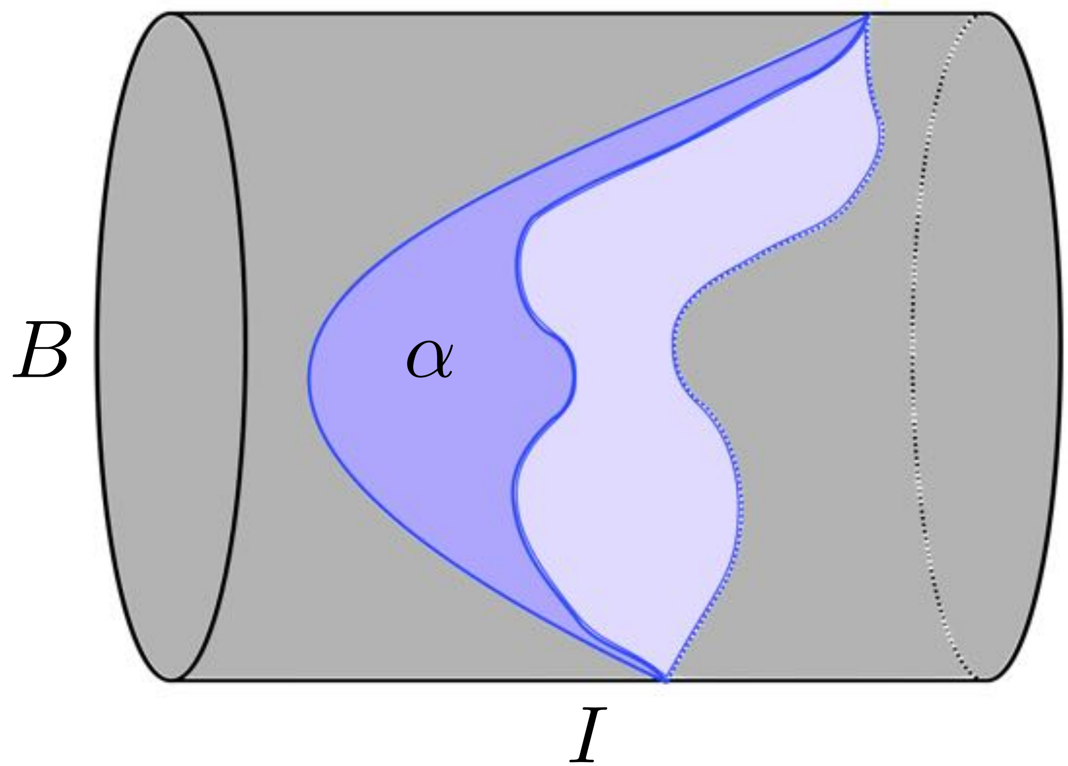

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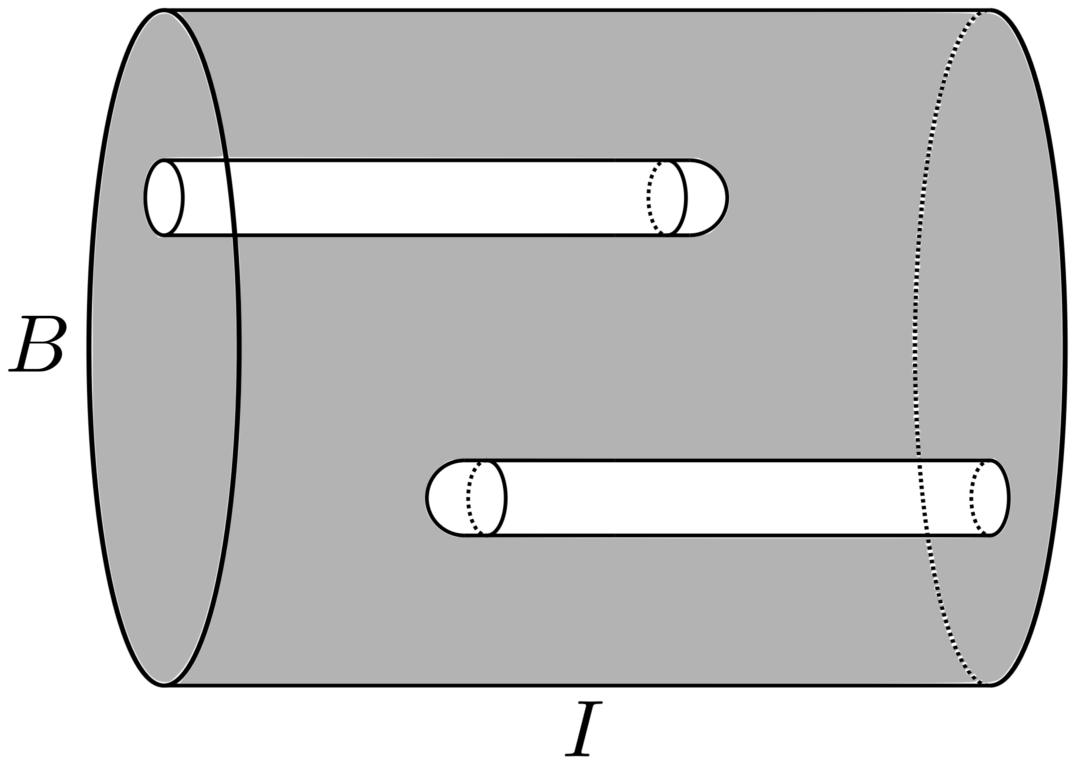

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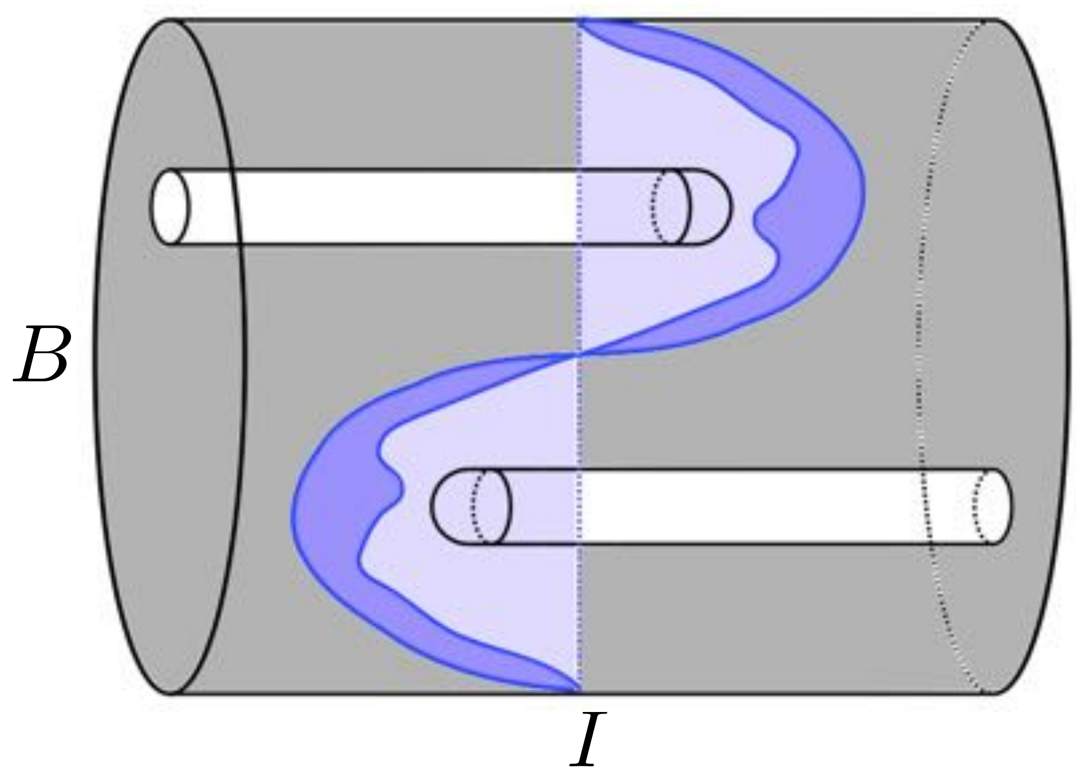

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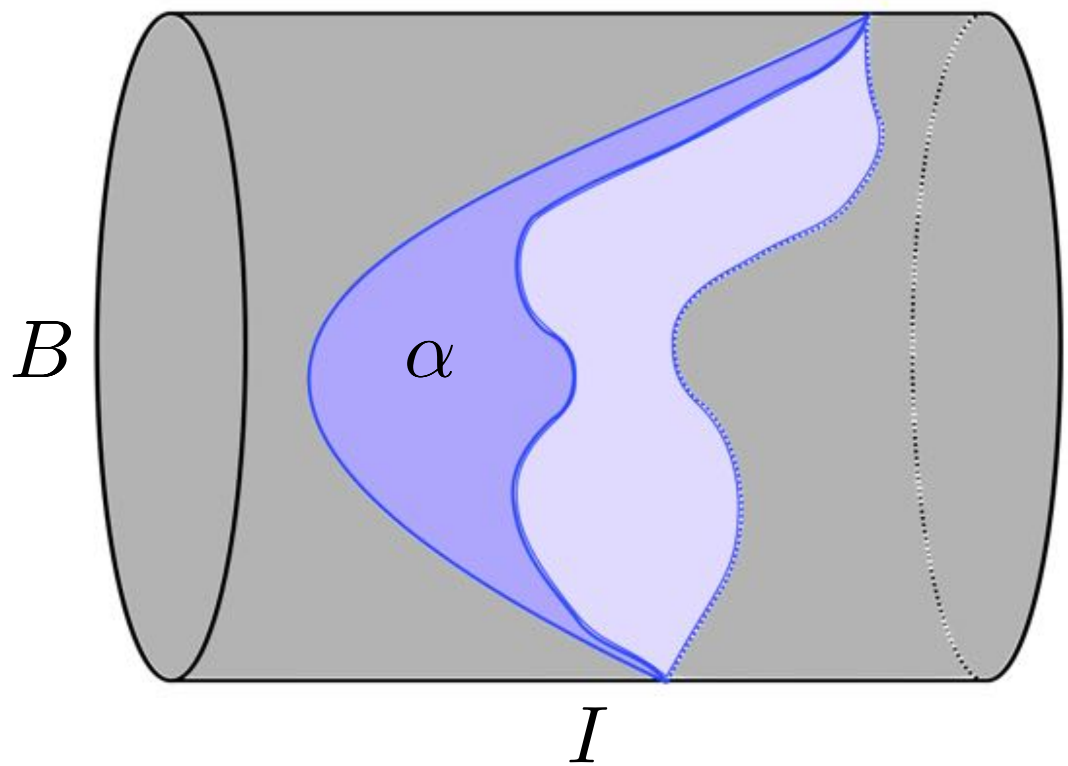

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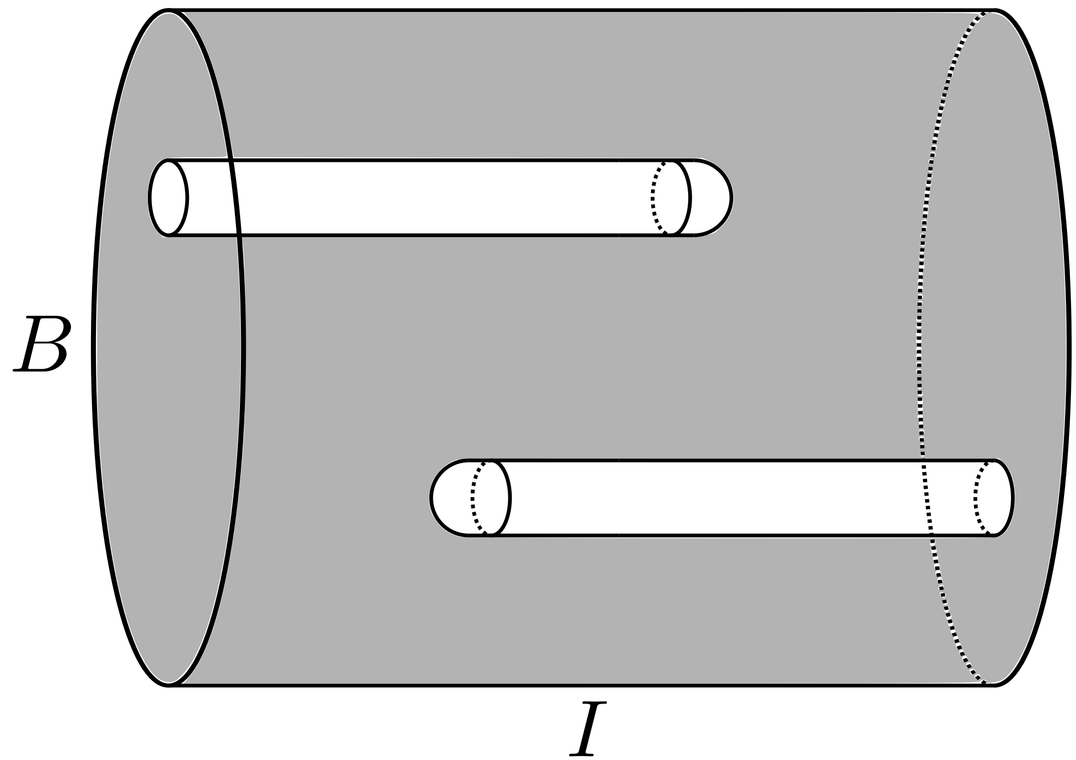

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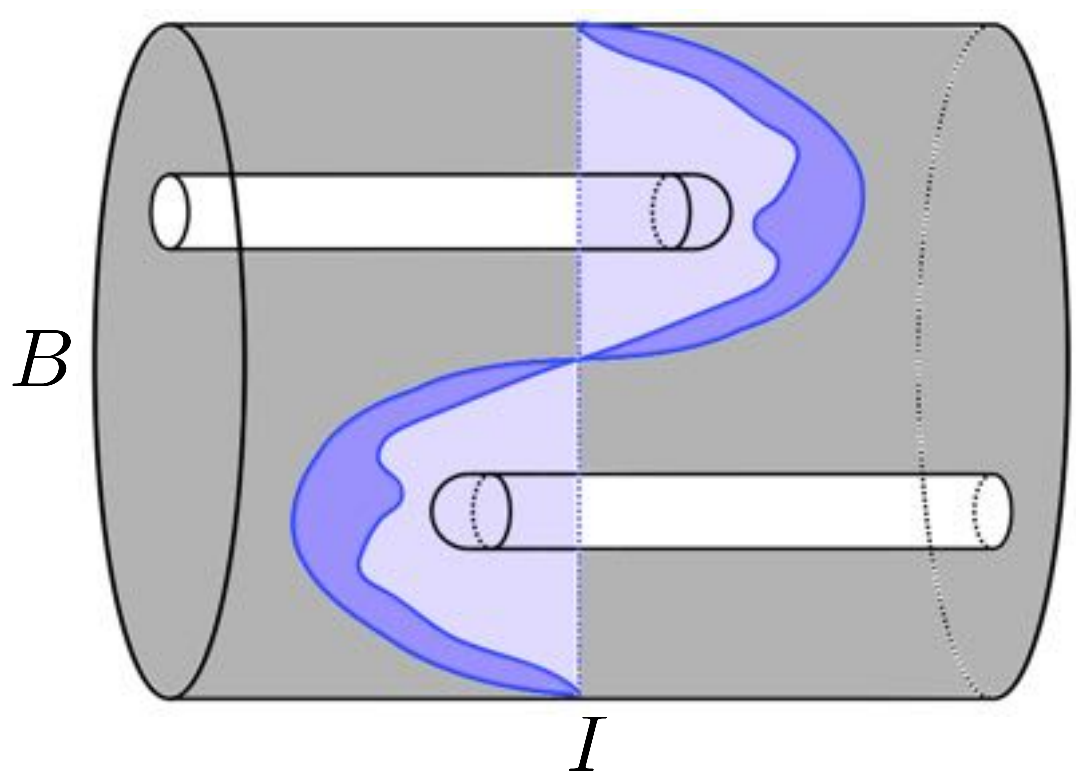

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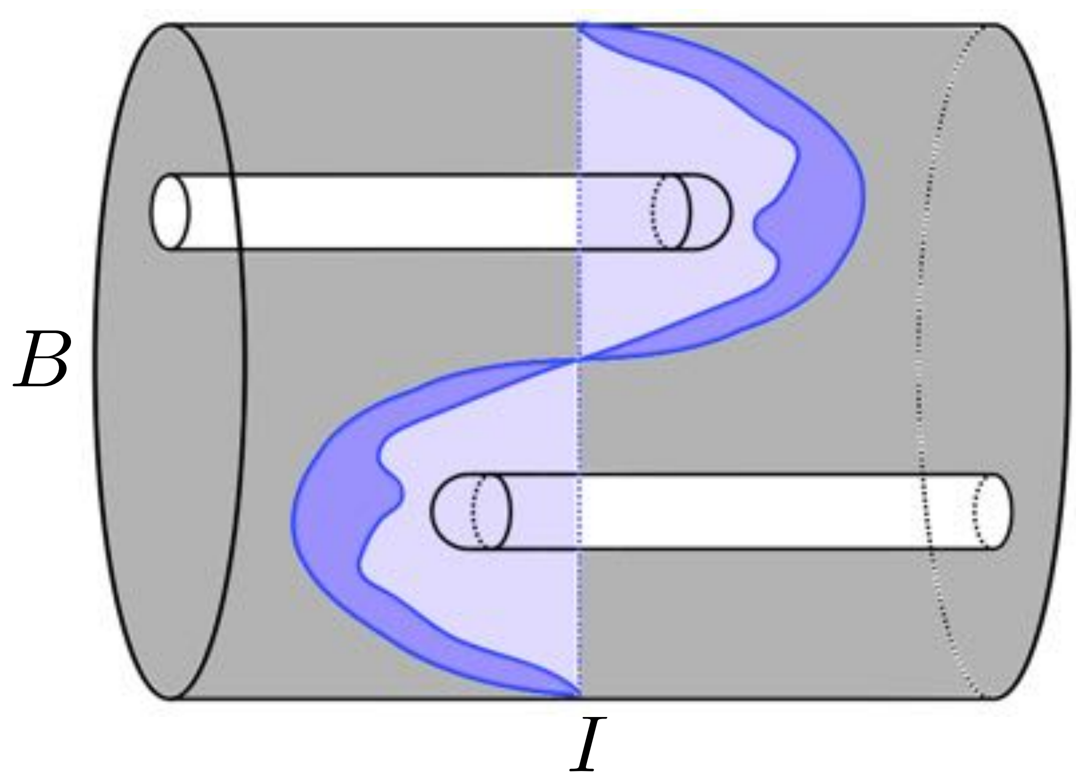

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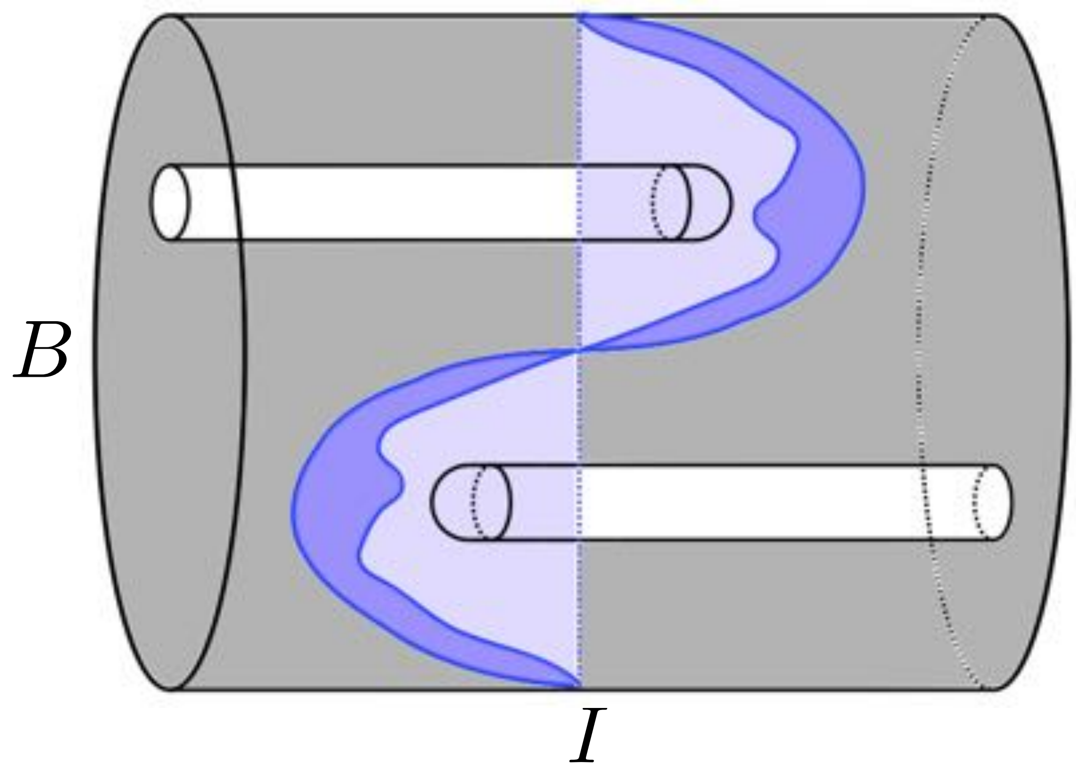

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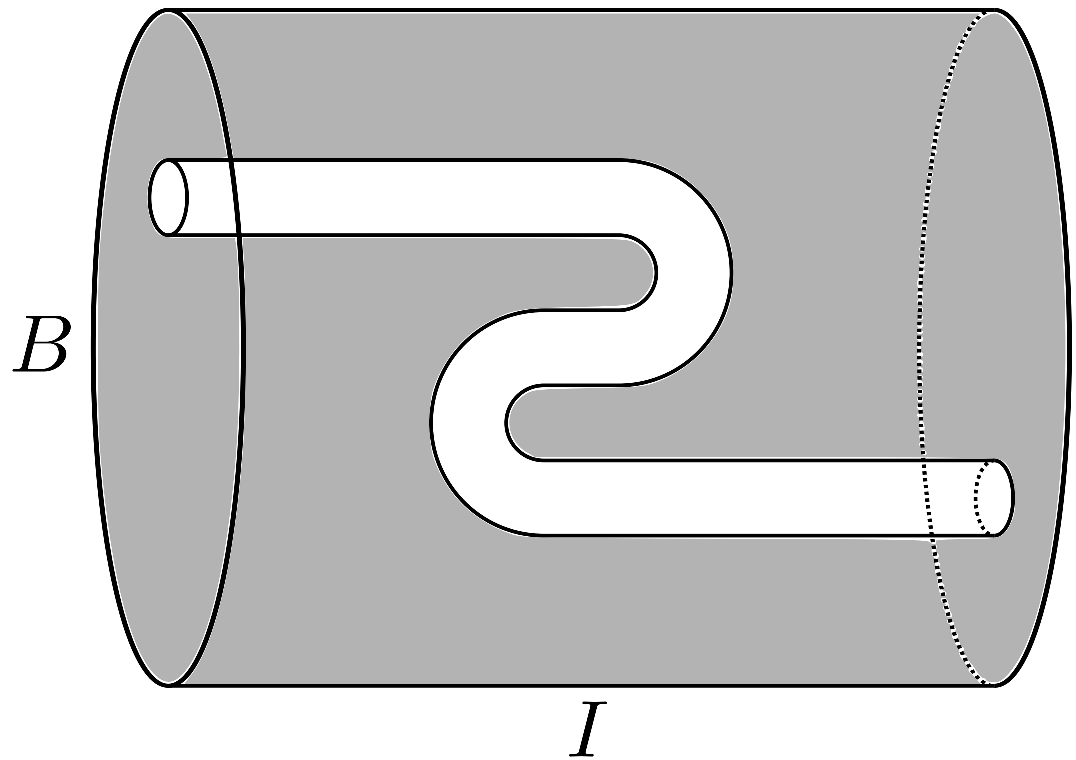

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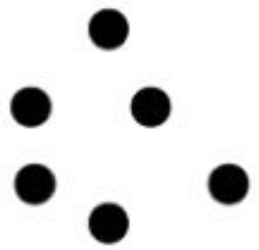
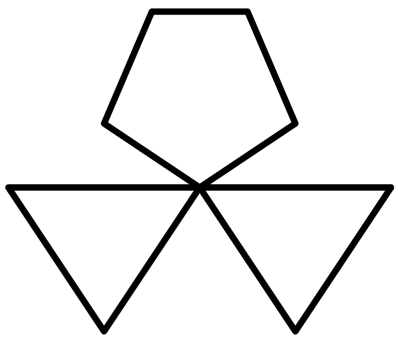
Homology

- *i*-dimensional homology "counts the number of *i*-dimensional holes"
- *i*-dimensional homology actually has the structure of a vector space!



0-dimensional homology: rank 6

1-dimensional homology: rank 0



0-dimensional homology: rank 1

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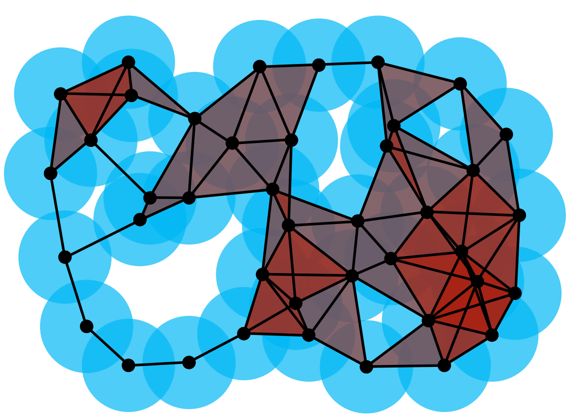

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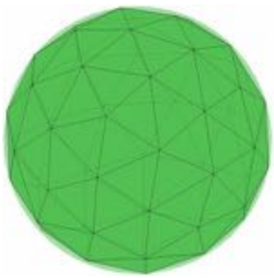

0-dimensional homology: rank 1

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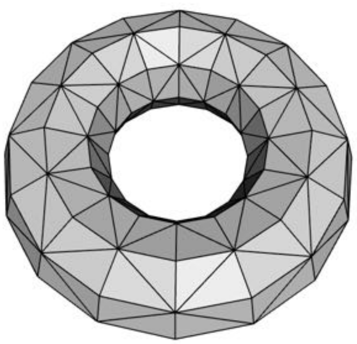
2-dimensional homology: rank 1



0-dimensional homology: rank 1

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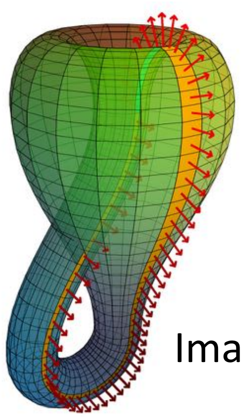
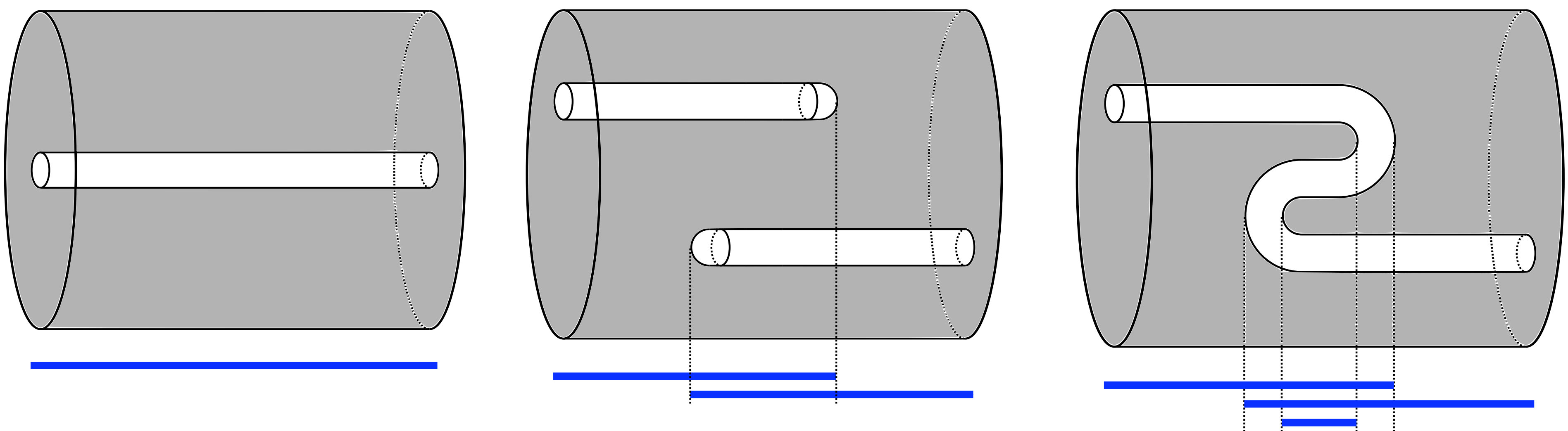


Be careful!

Image credit: https://plus.maths.org/content/imaging-maths-inside-klein-bottle

Zigzag persistent homology

Form zigzag module for $X \longrightarrow I$ with $(d-1)$–dimensional homology.



Zigzag Persistence by G. Carlsson and V. de Silva

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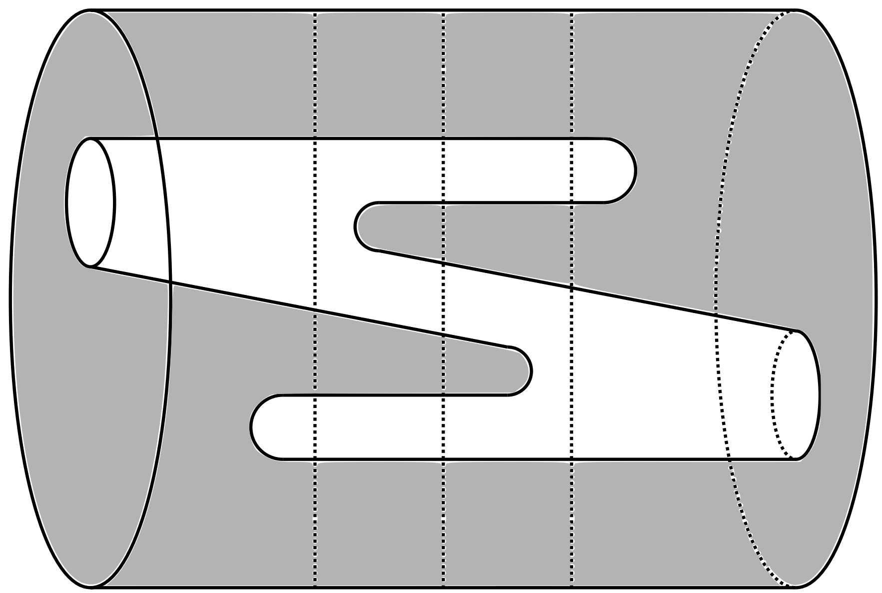

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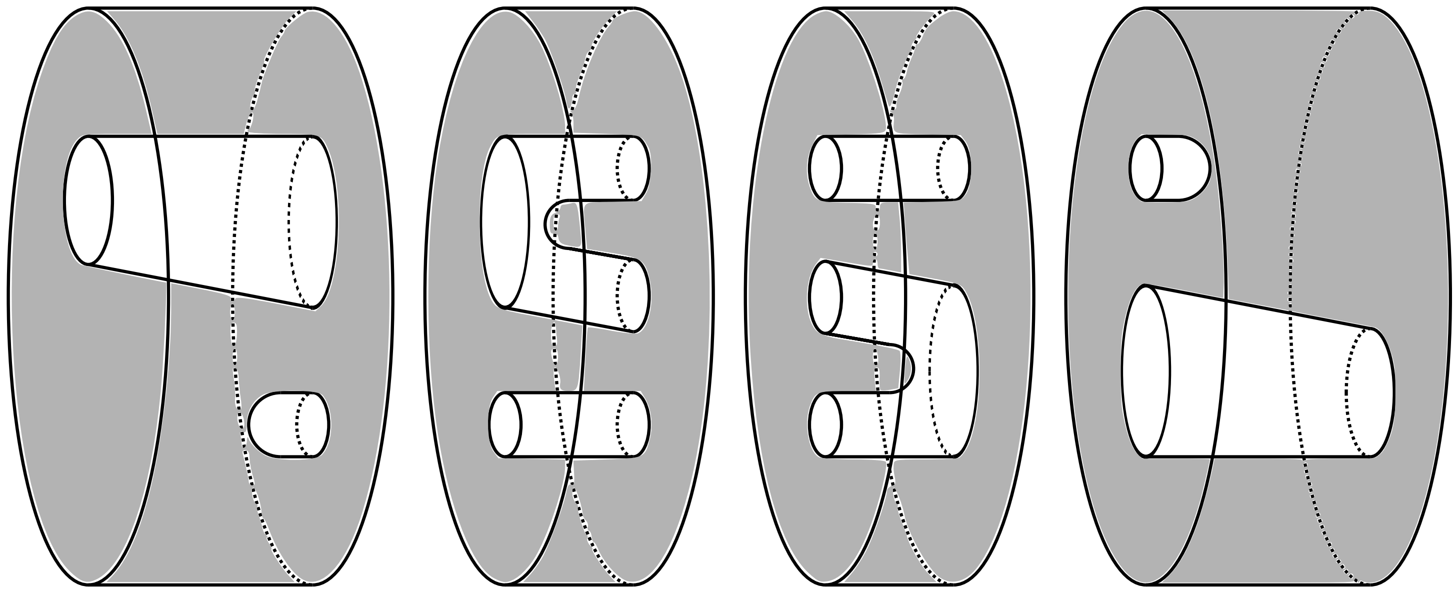

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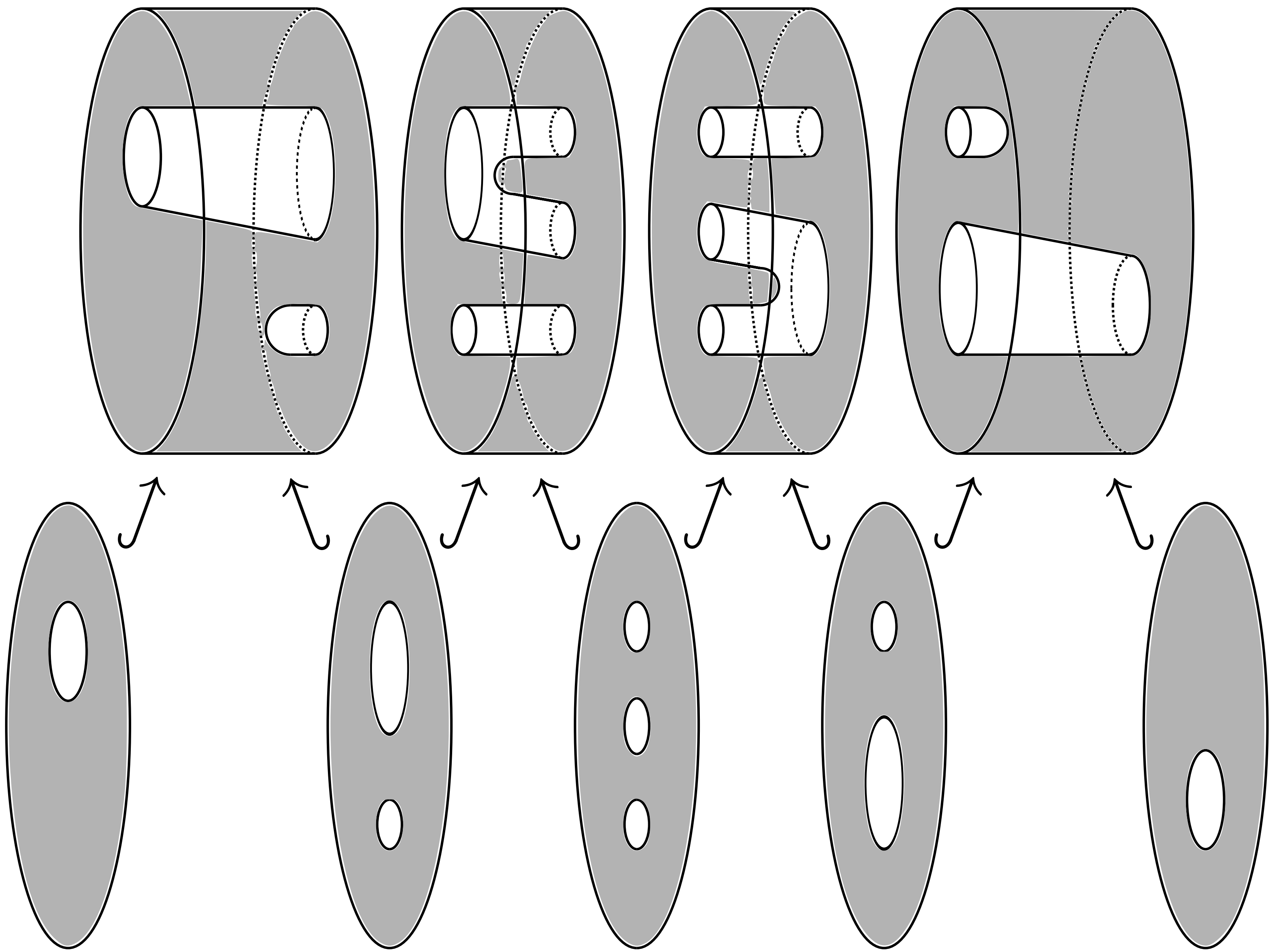

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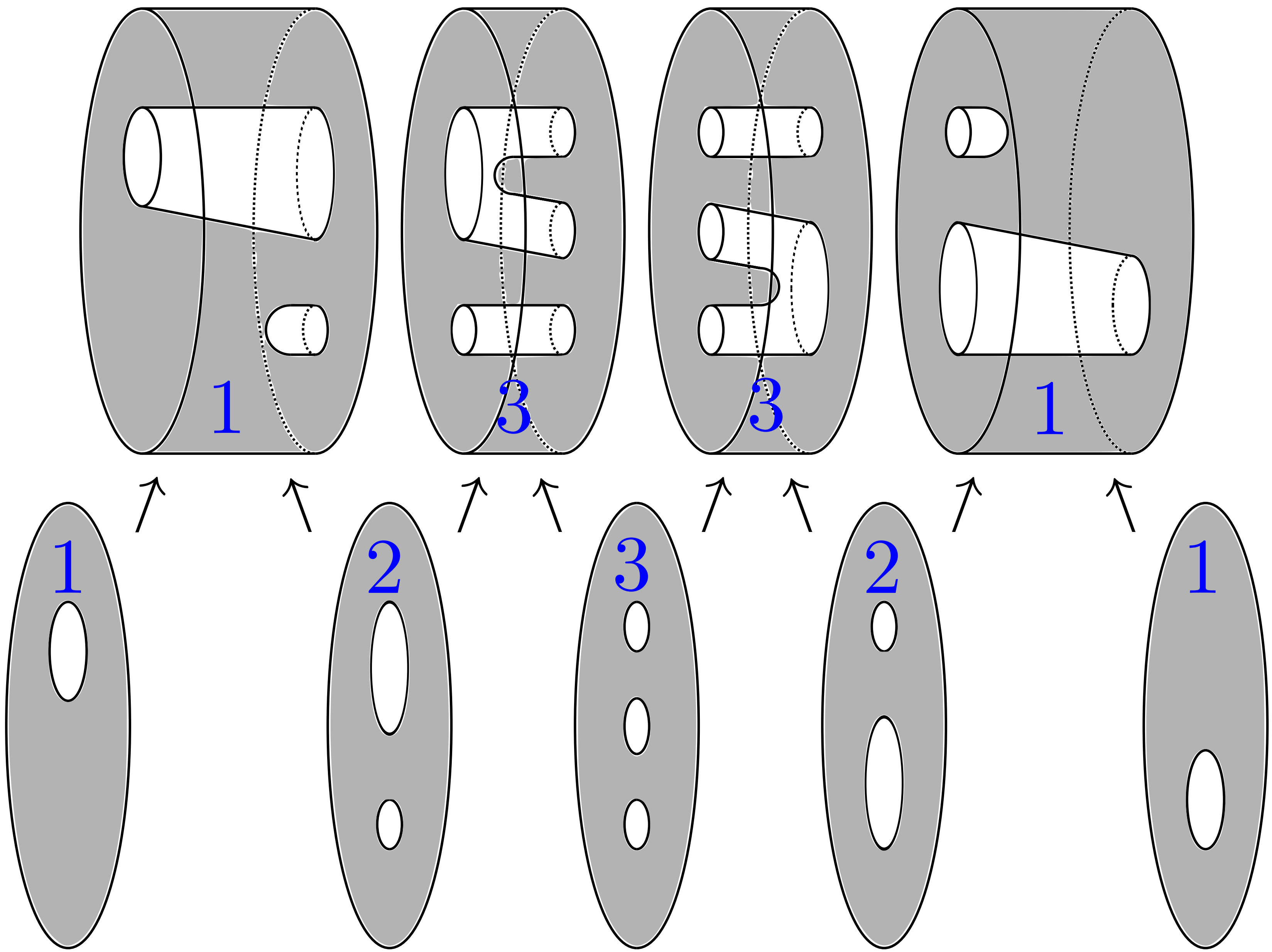

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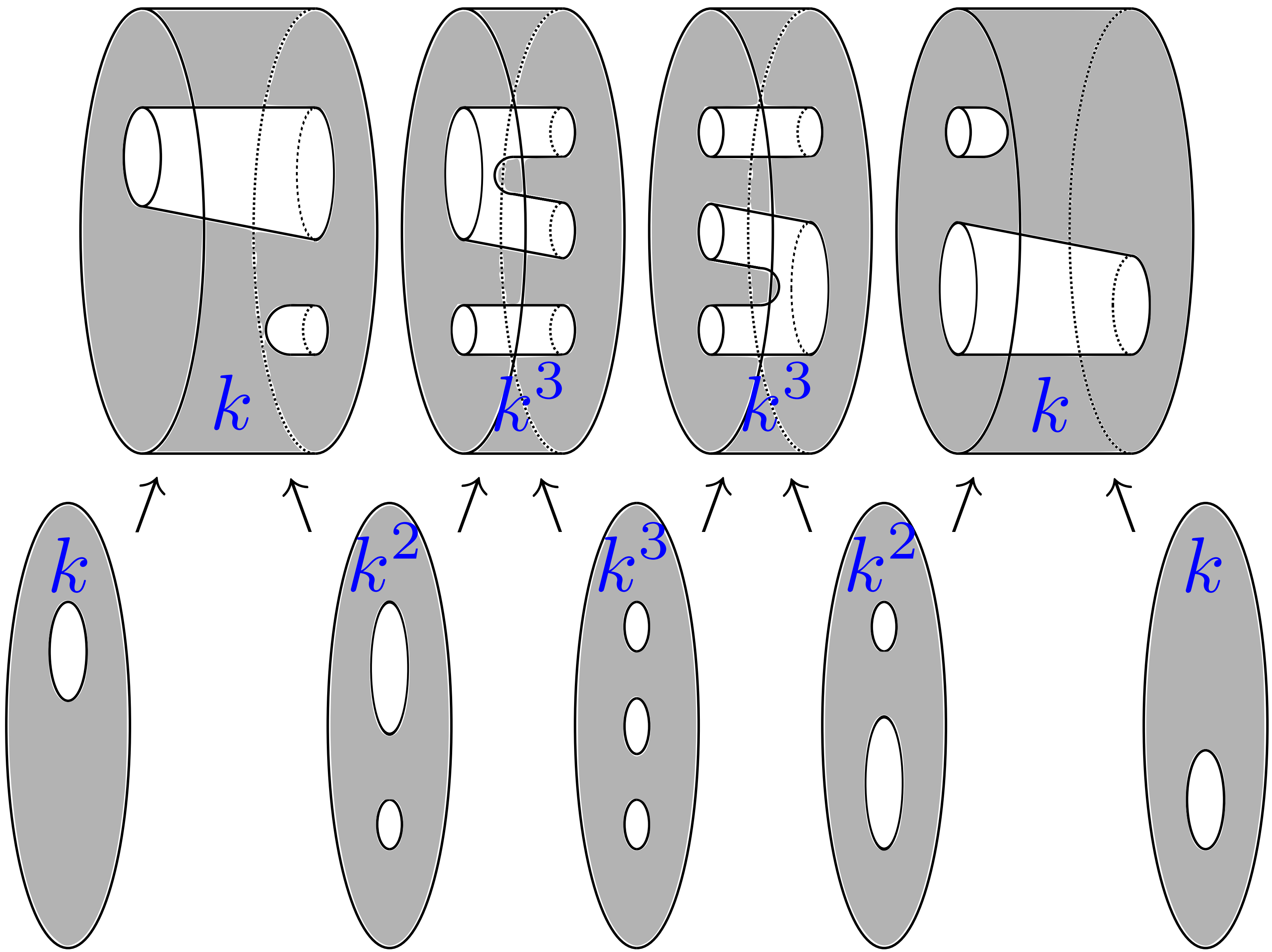

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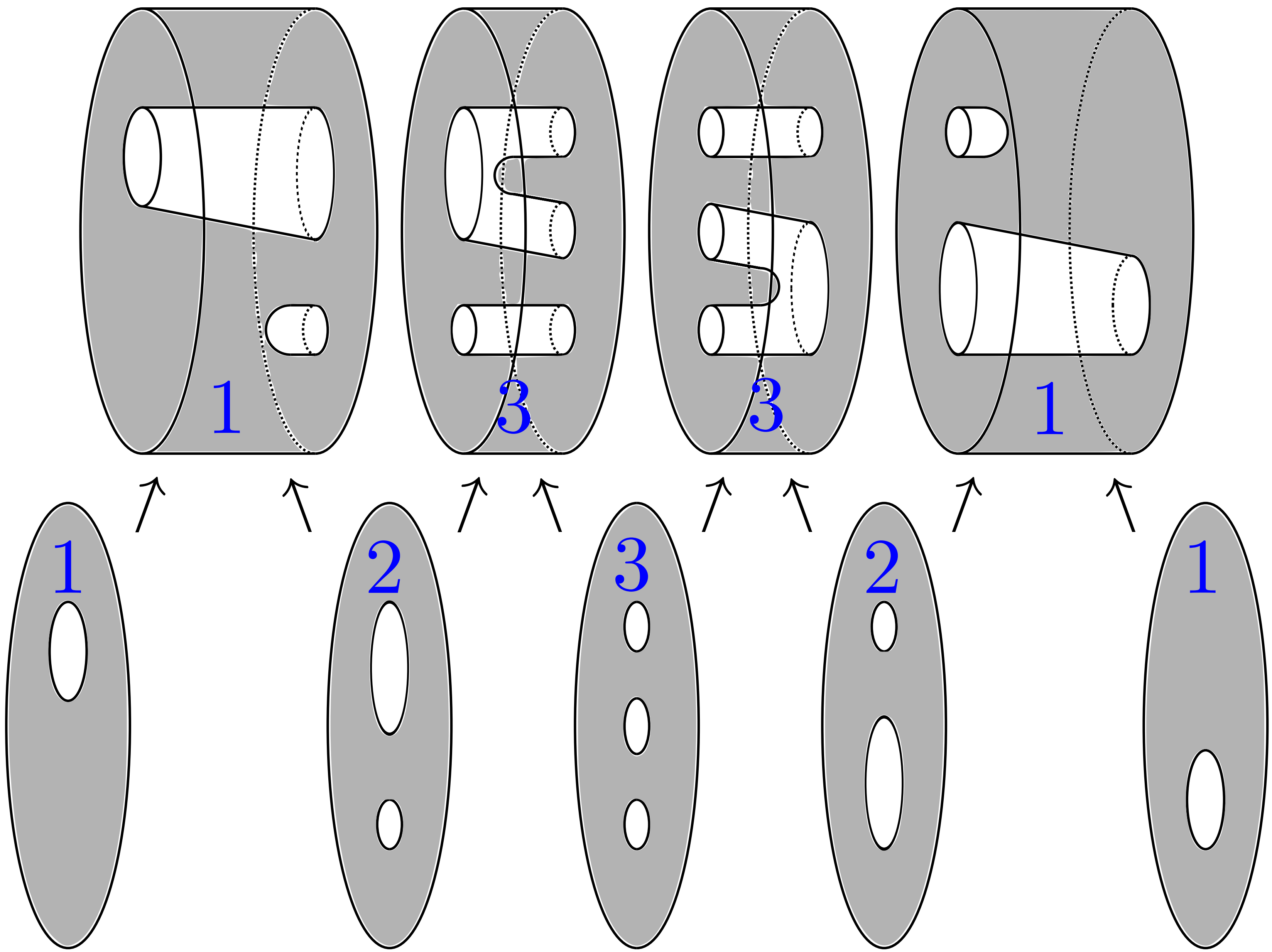

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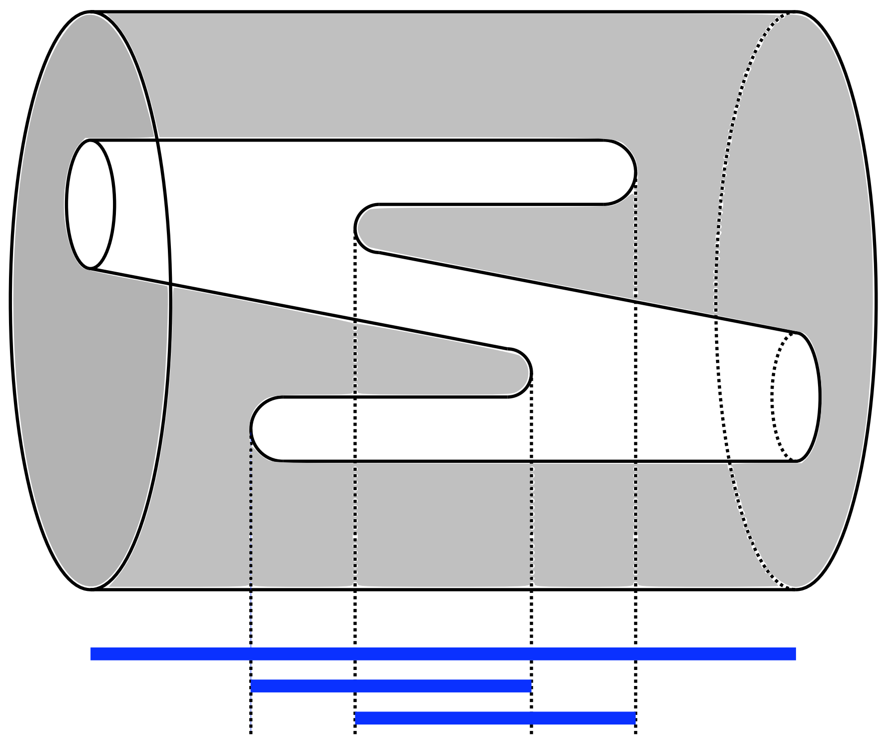

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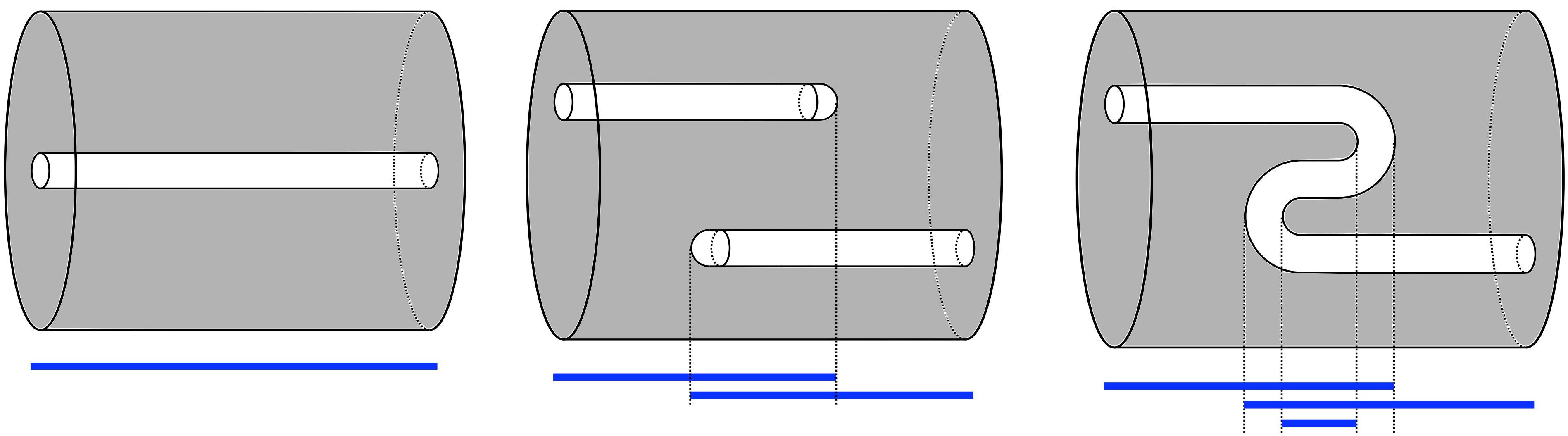

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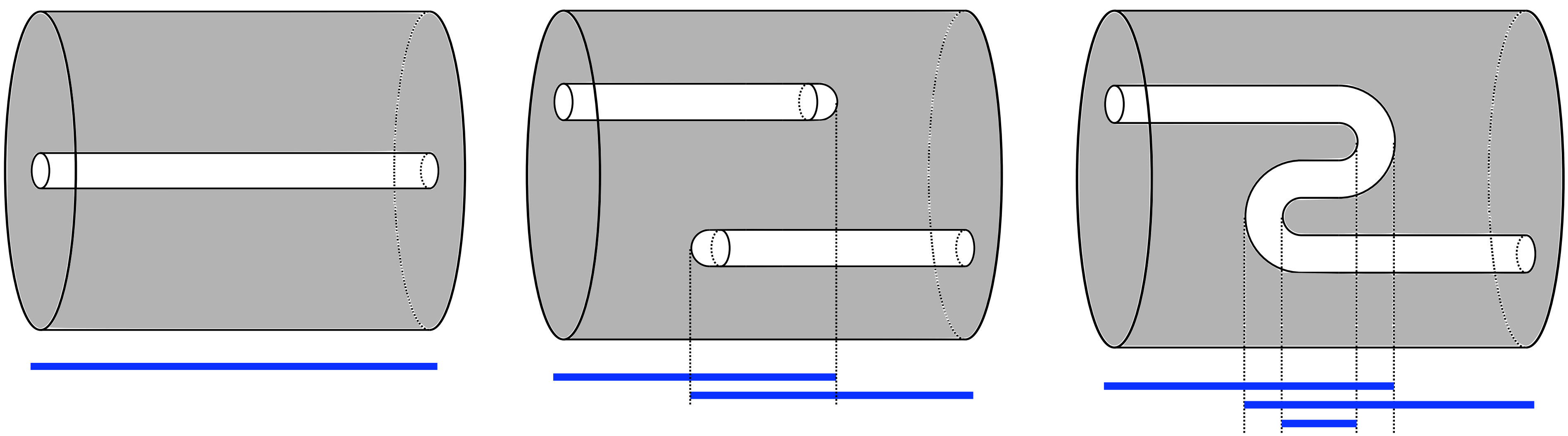

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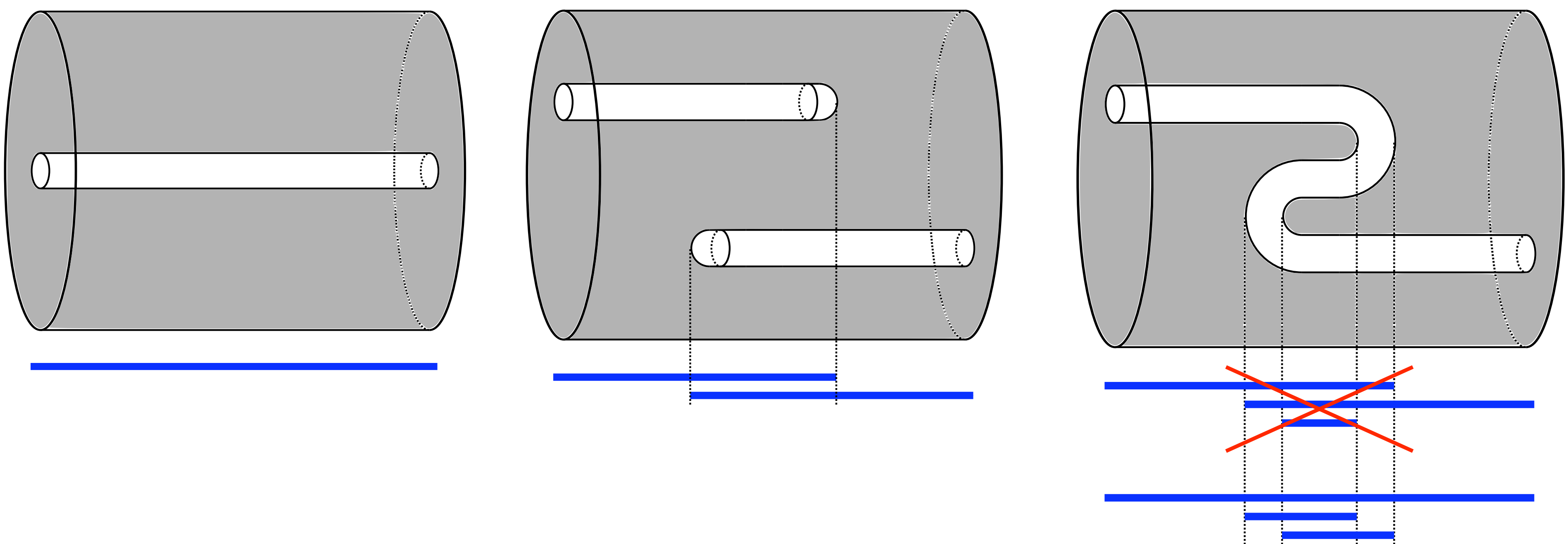

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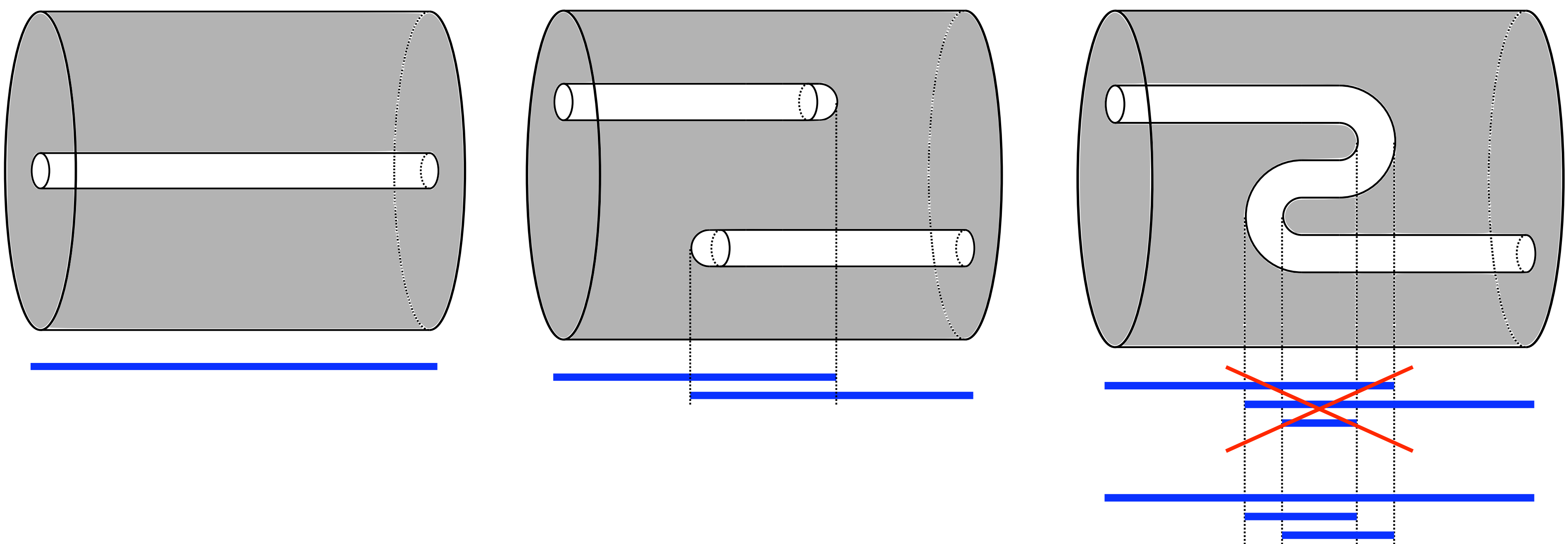

- <u>Theorem.</u>

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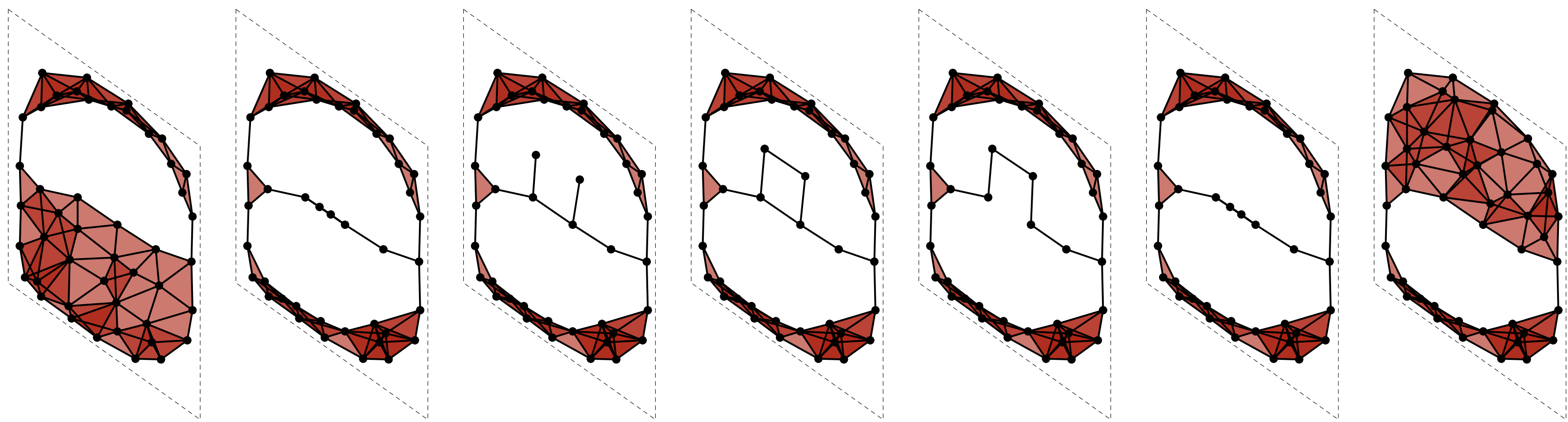
- Streaming computation.

Dependence on embedding $X \hookrightarrow B \times I$

- The time-varying Čech complex of X does not determine if an evasion path exists!



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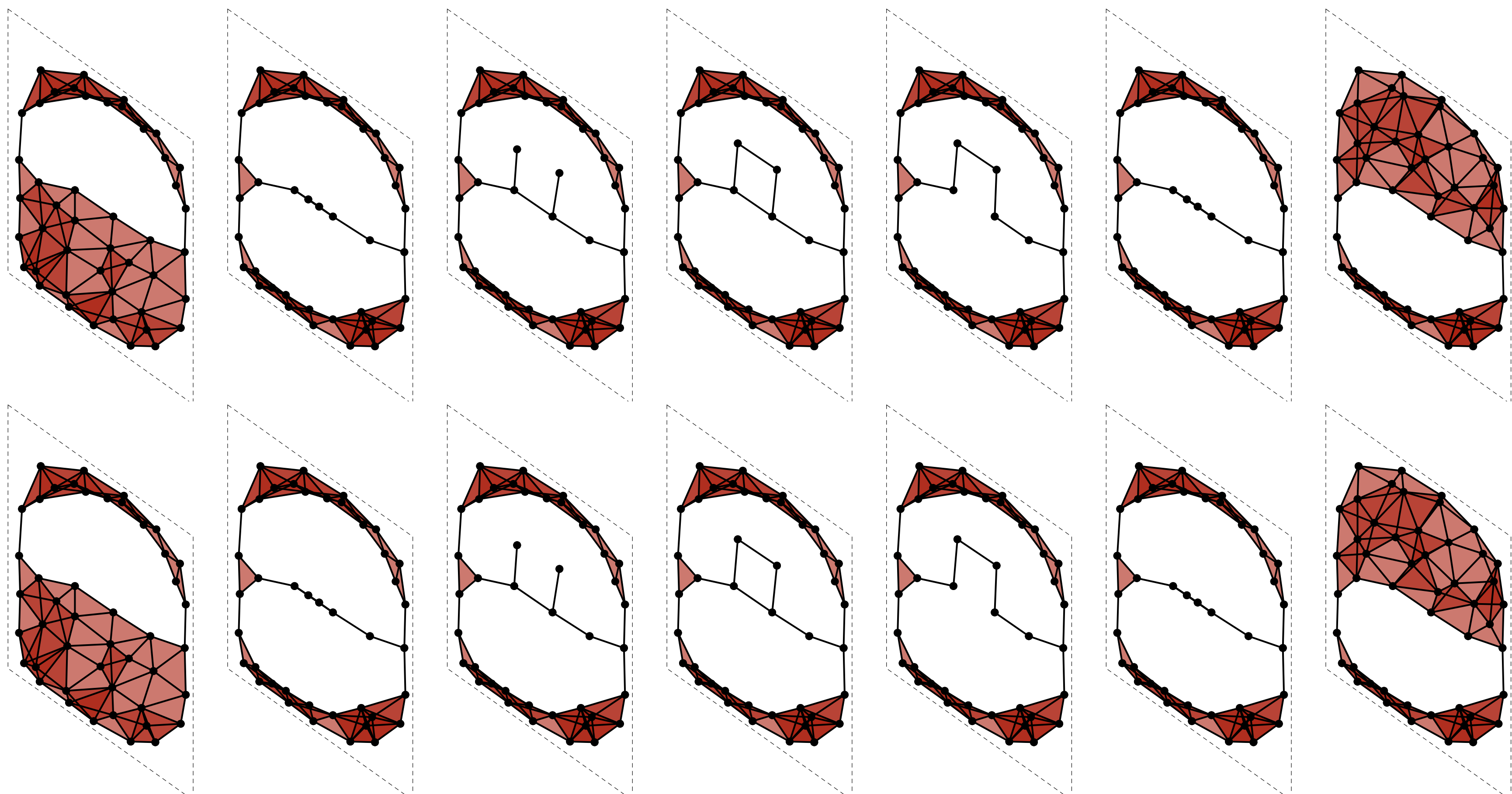

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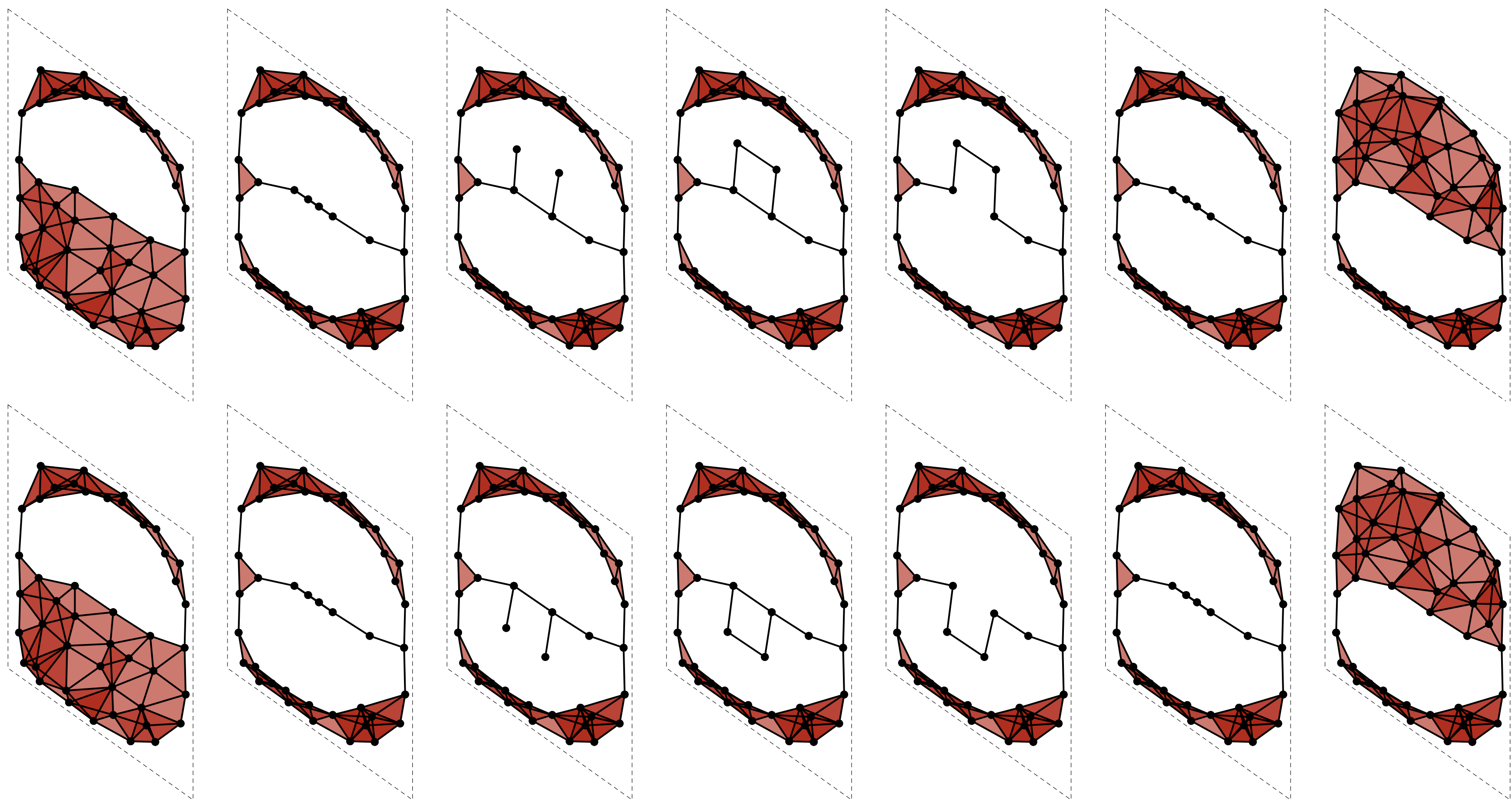

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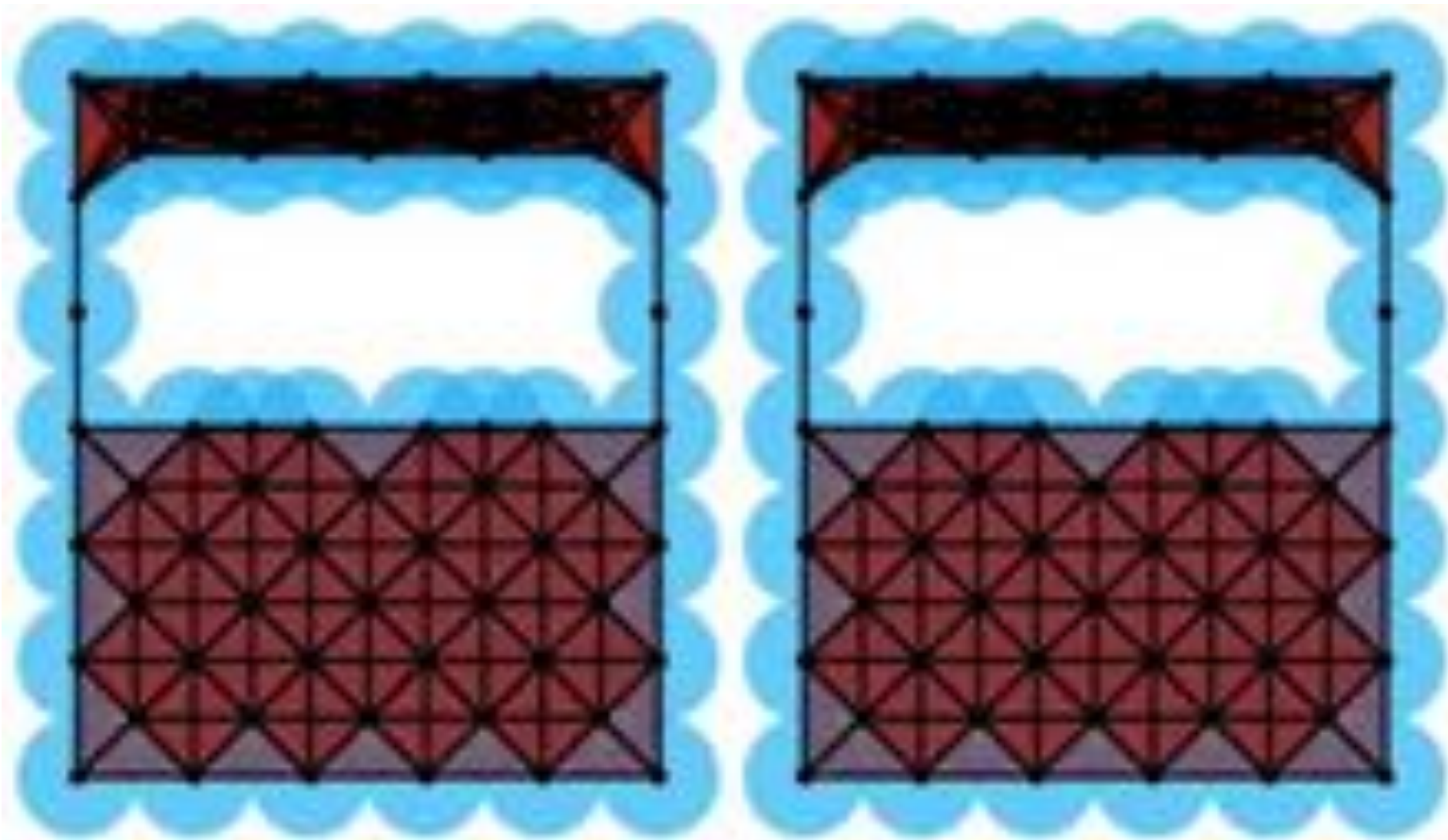

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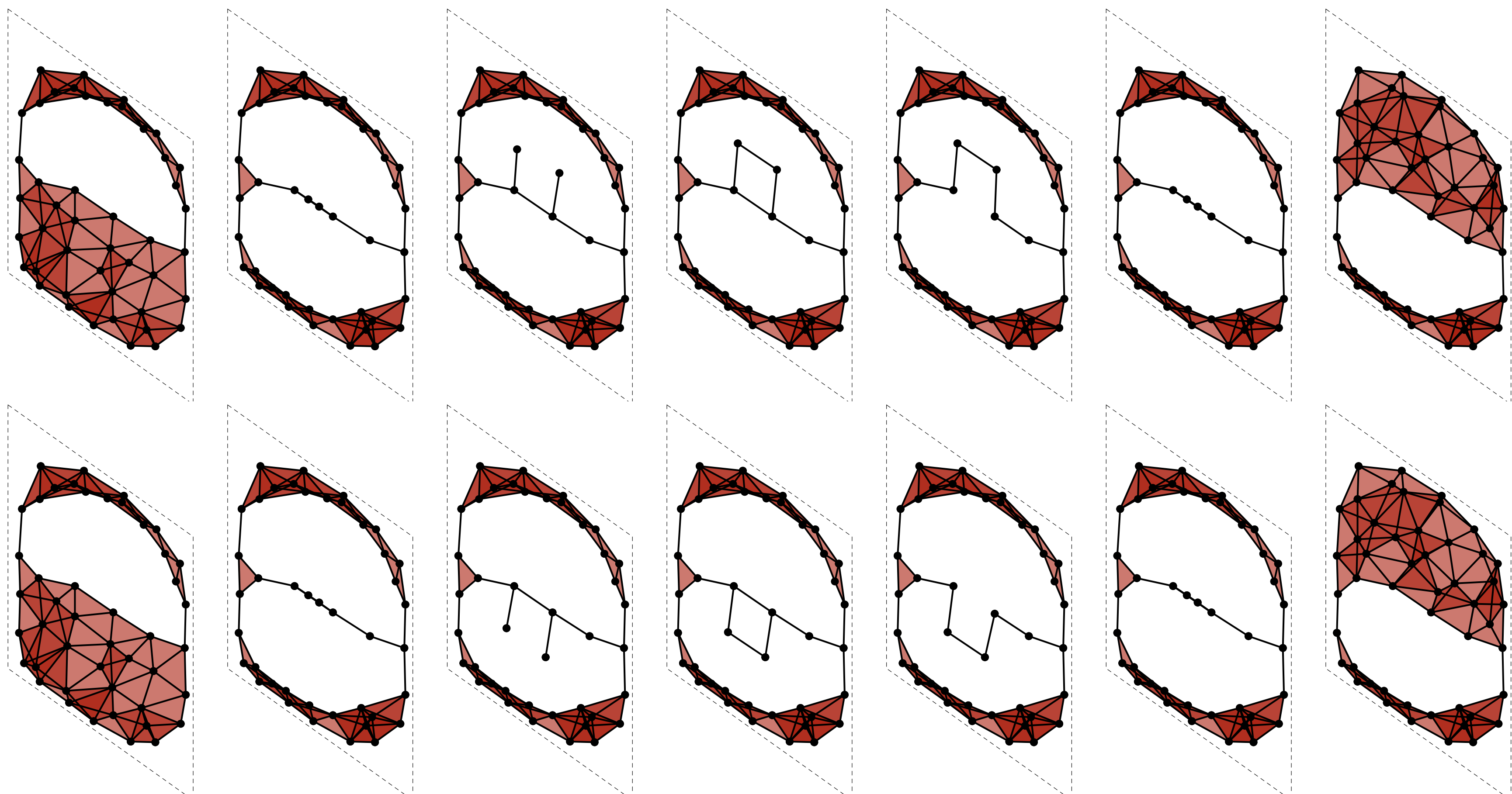

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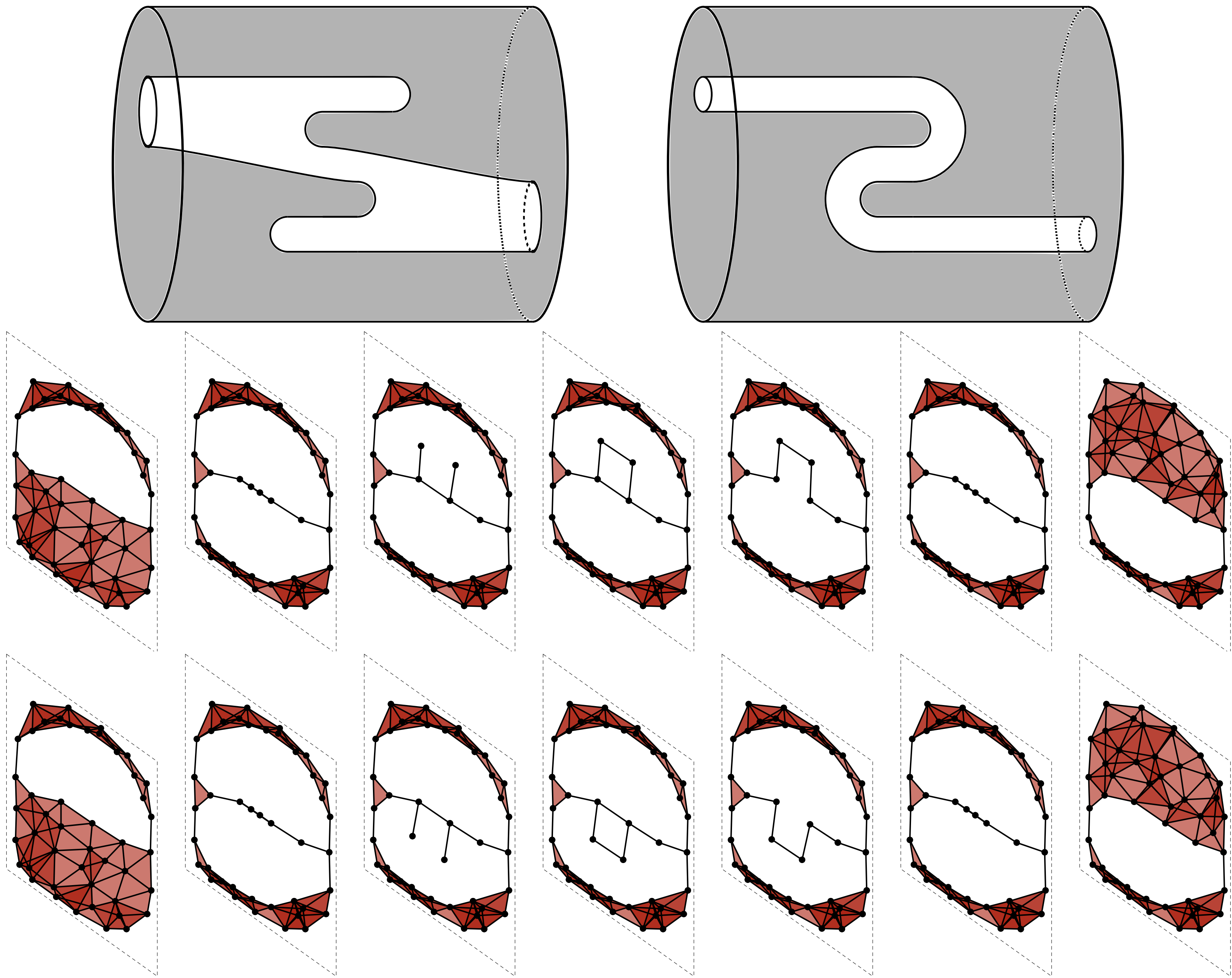

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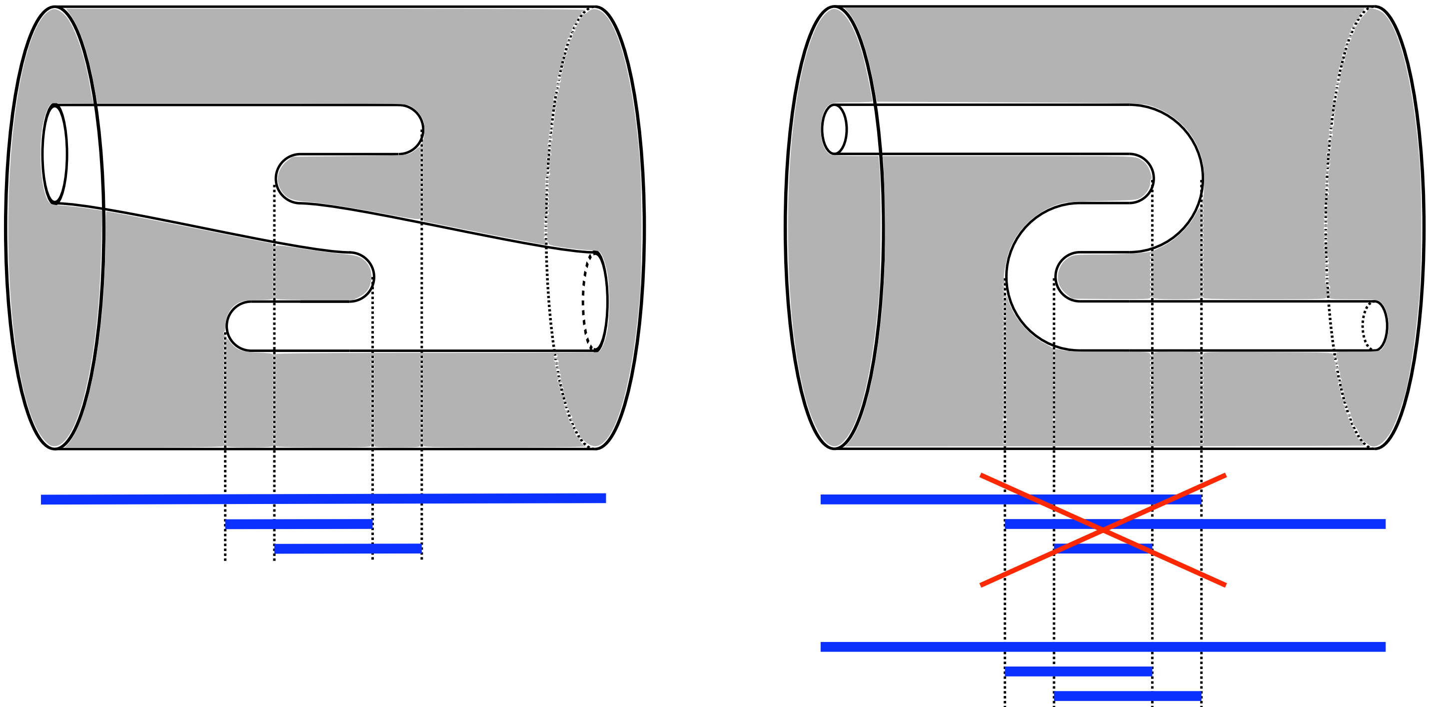
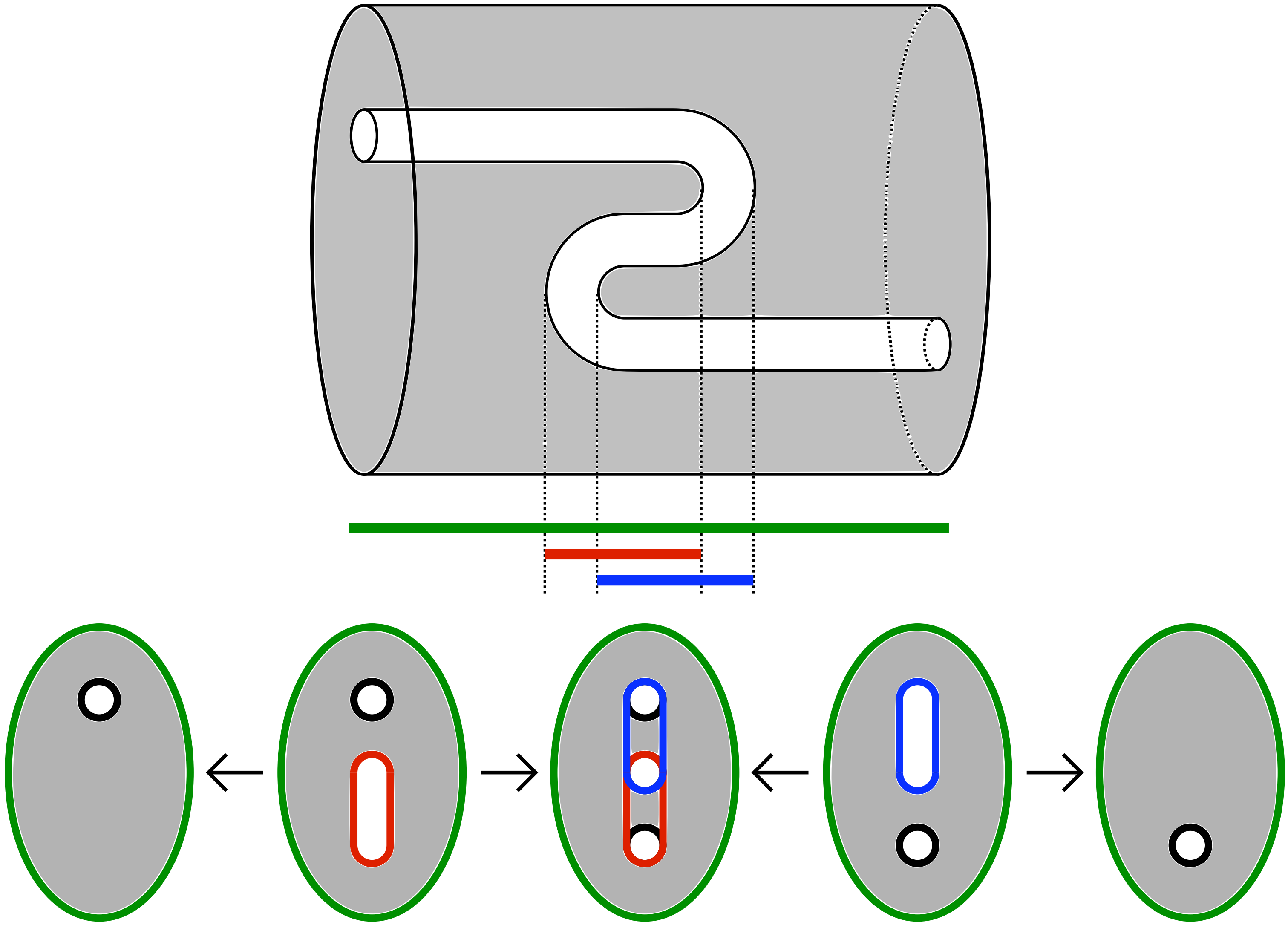


- The two covered regions are "topologically indistinguishable in a time-preserving way", but the uncovered regions are not!

Zigzag persistence

- <u>Caution 2.9 of *Zigzag Persistence*.</u> Not every submodule isomorphic to an interval corresponds to a summand.



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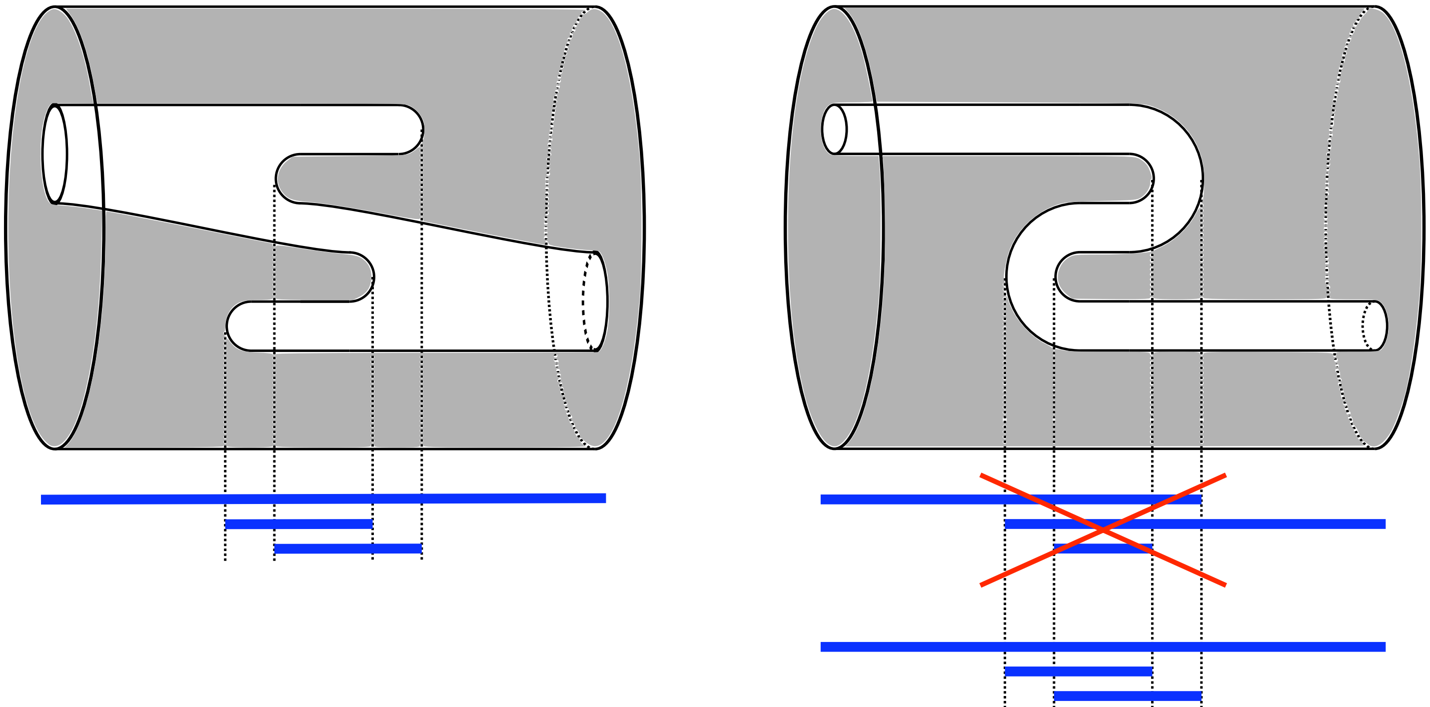

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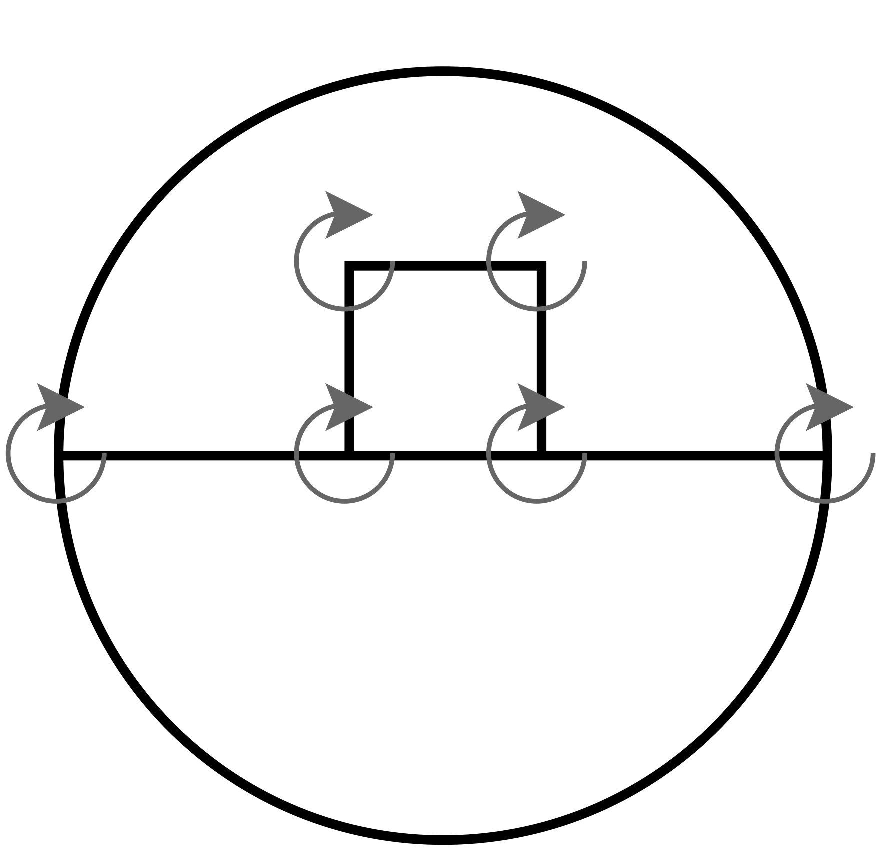
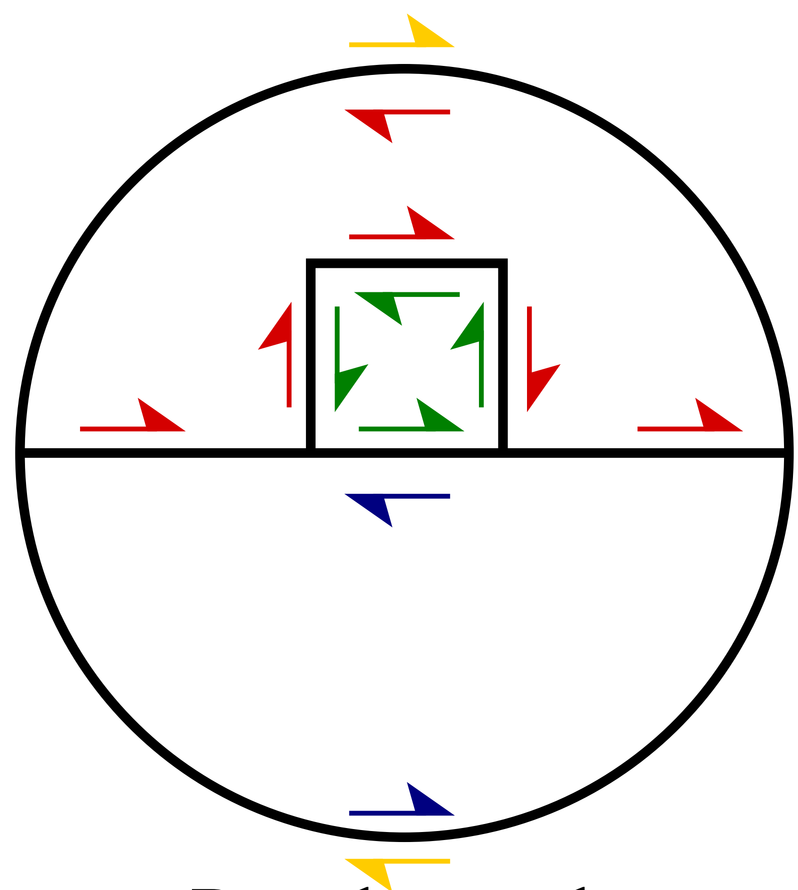
Fat graphs

- A fat graph structure specifies a cyclic ordering of edges about each vertex (left).
- Equivalent to a set of boundary cycles (right).



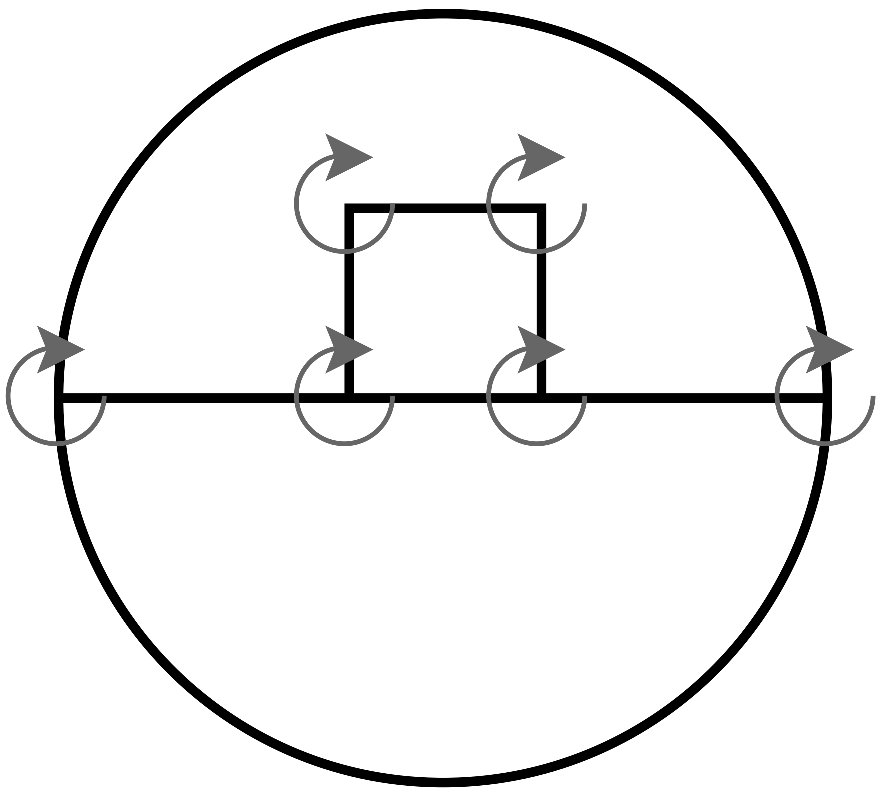
Cyclic orderings



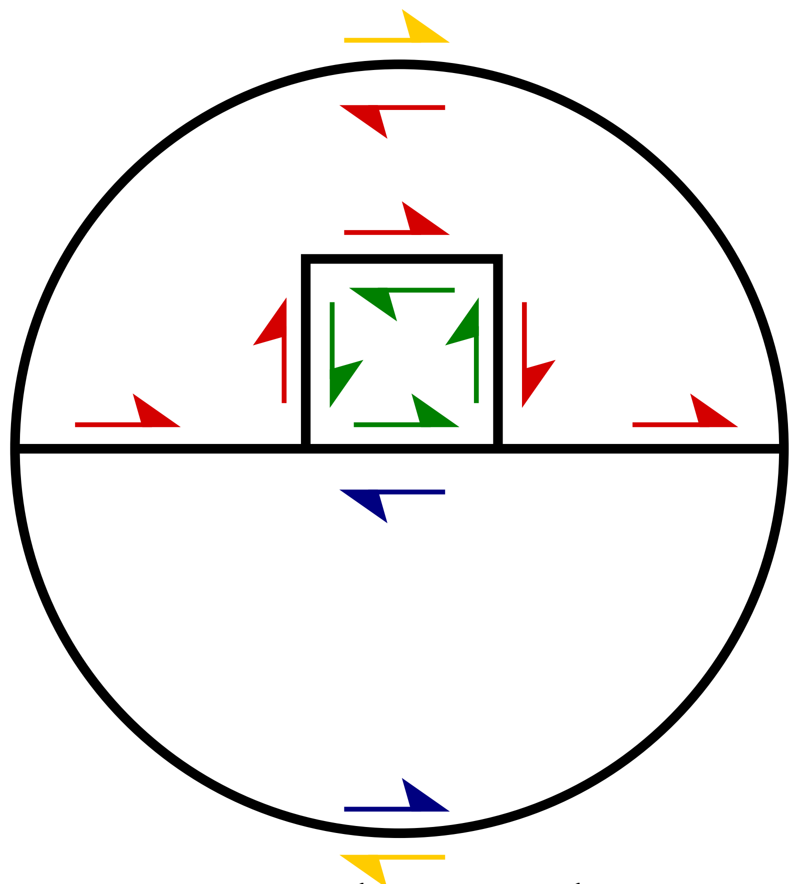
Boundary cycles

Planar sensors measuring cyclic orders

- Theorem. In a planar sensor network that remains connected, the time-varying alpha complex with rotation information determines if an evasion path exists.



Cyclic orderings



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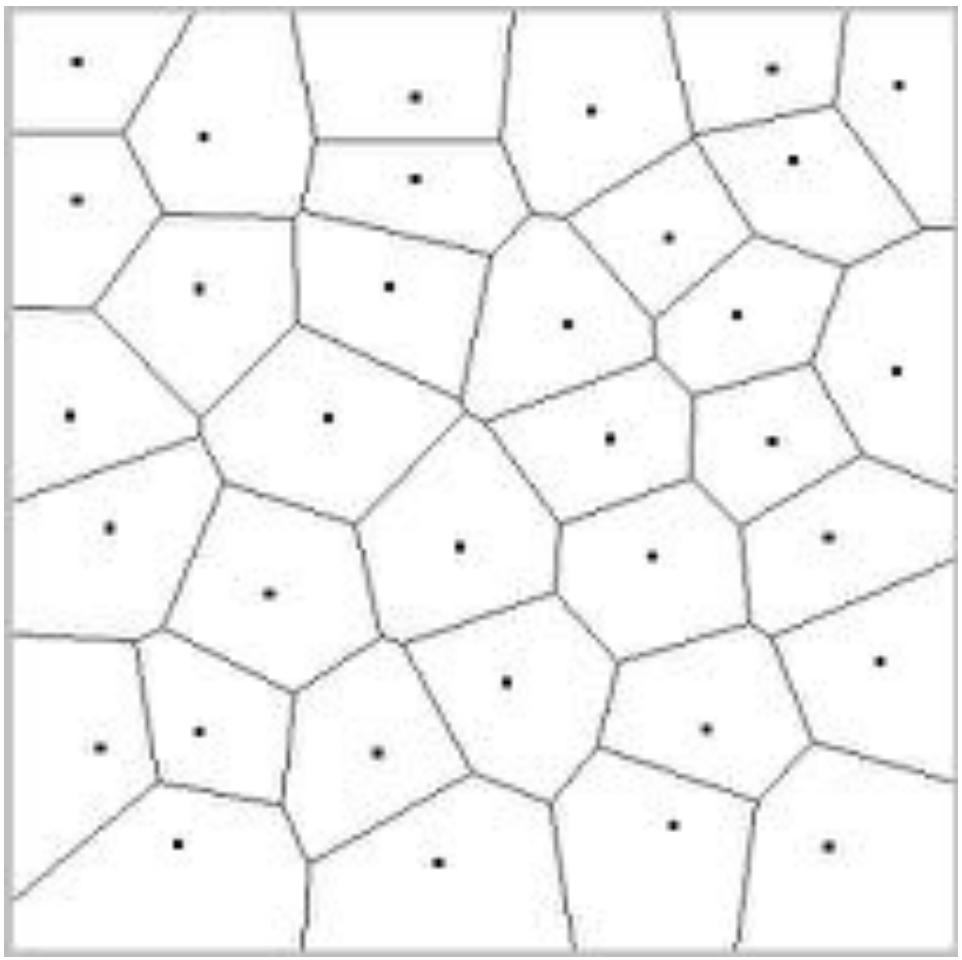
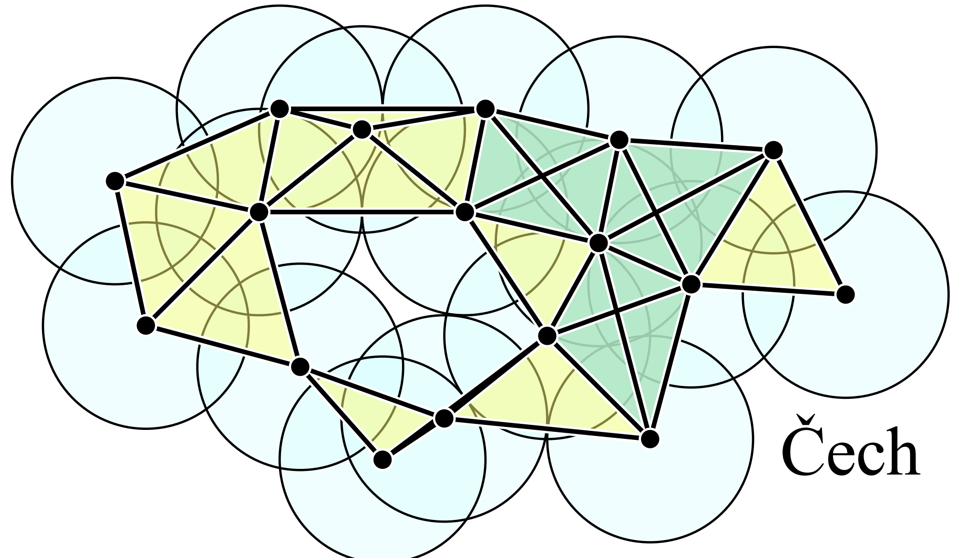
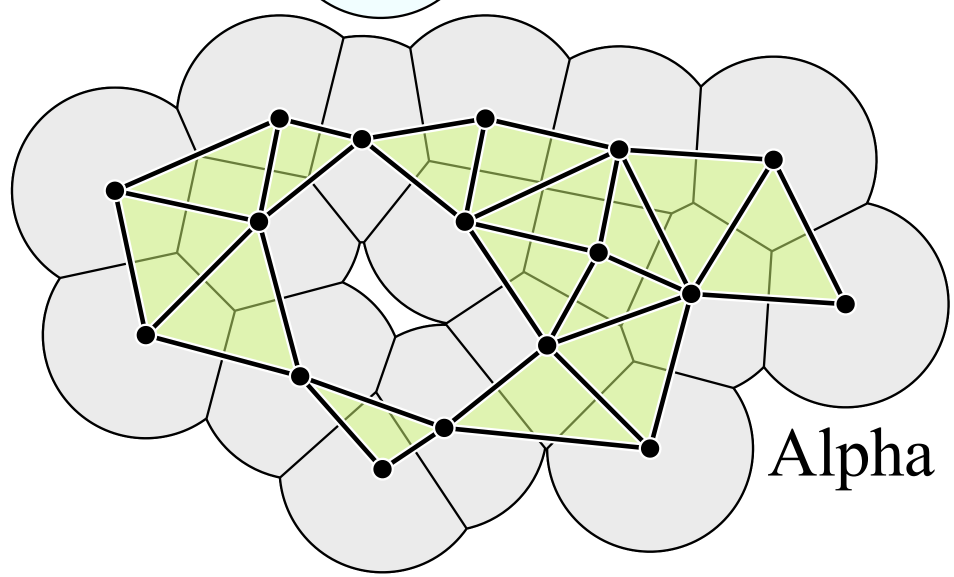


Voronoi regions



Čech



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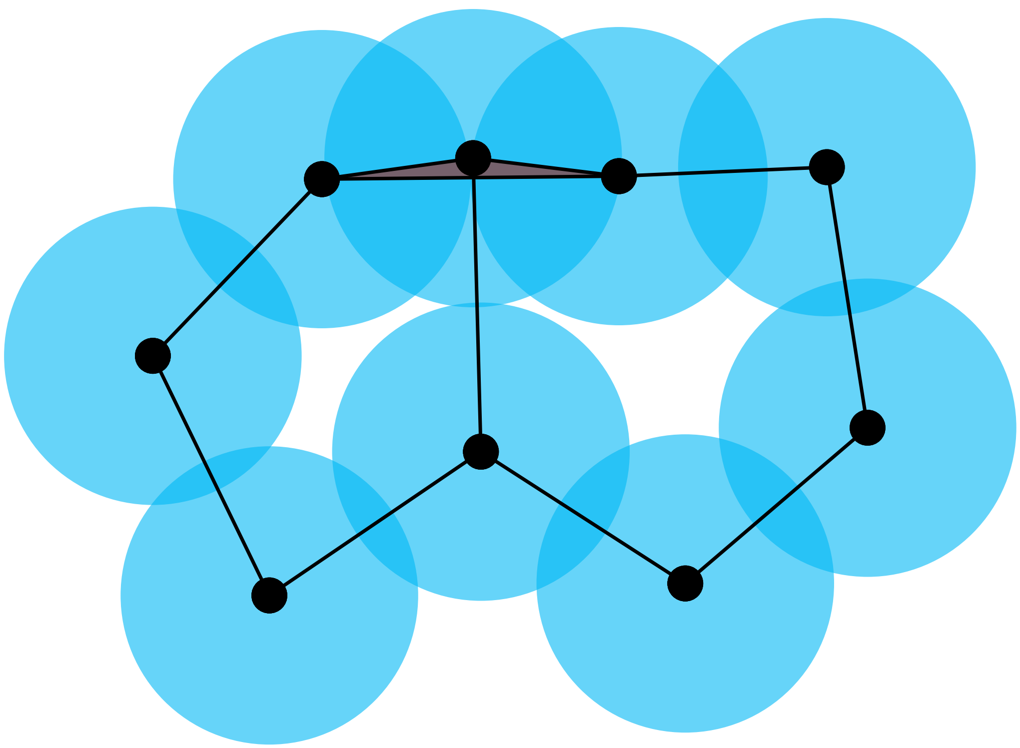

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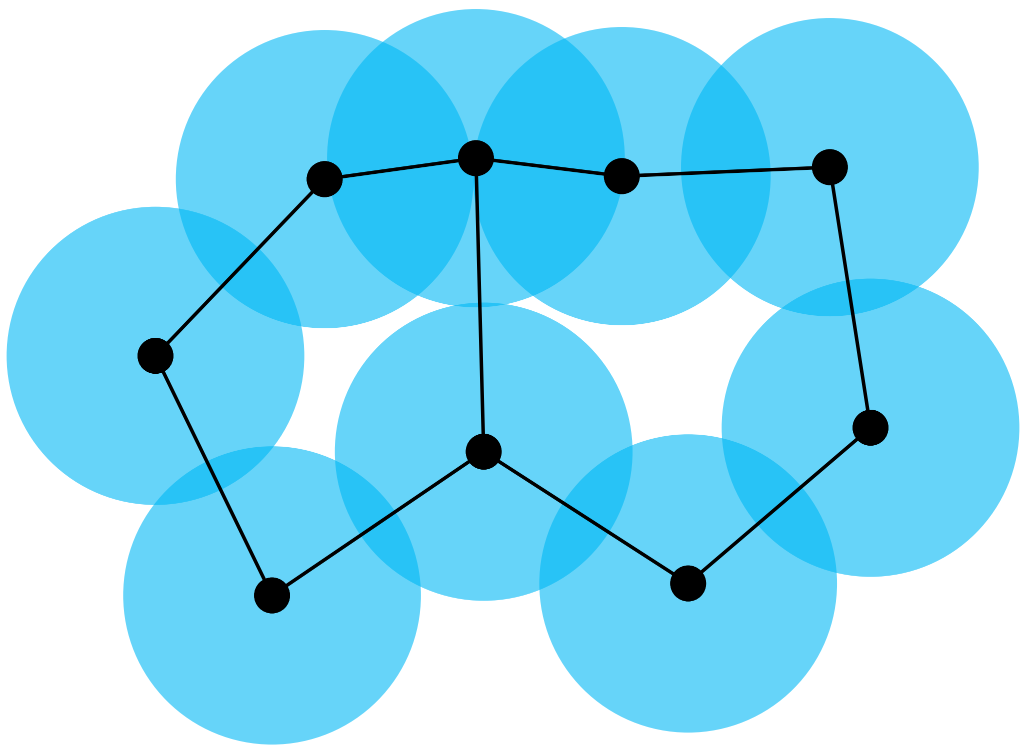

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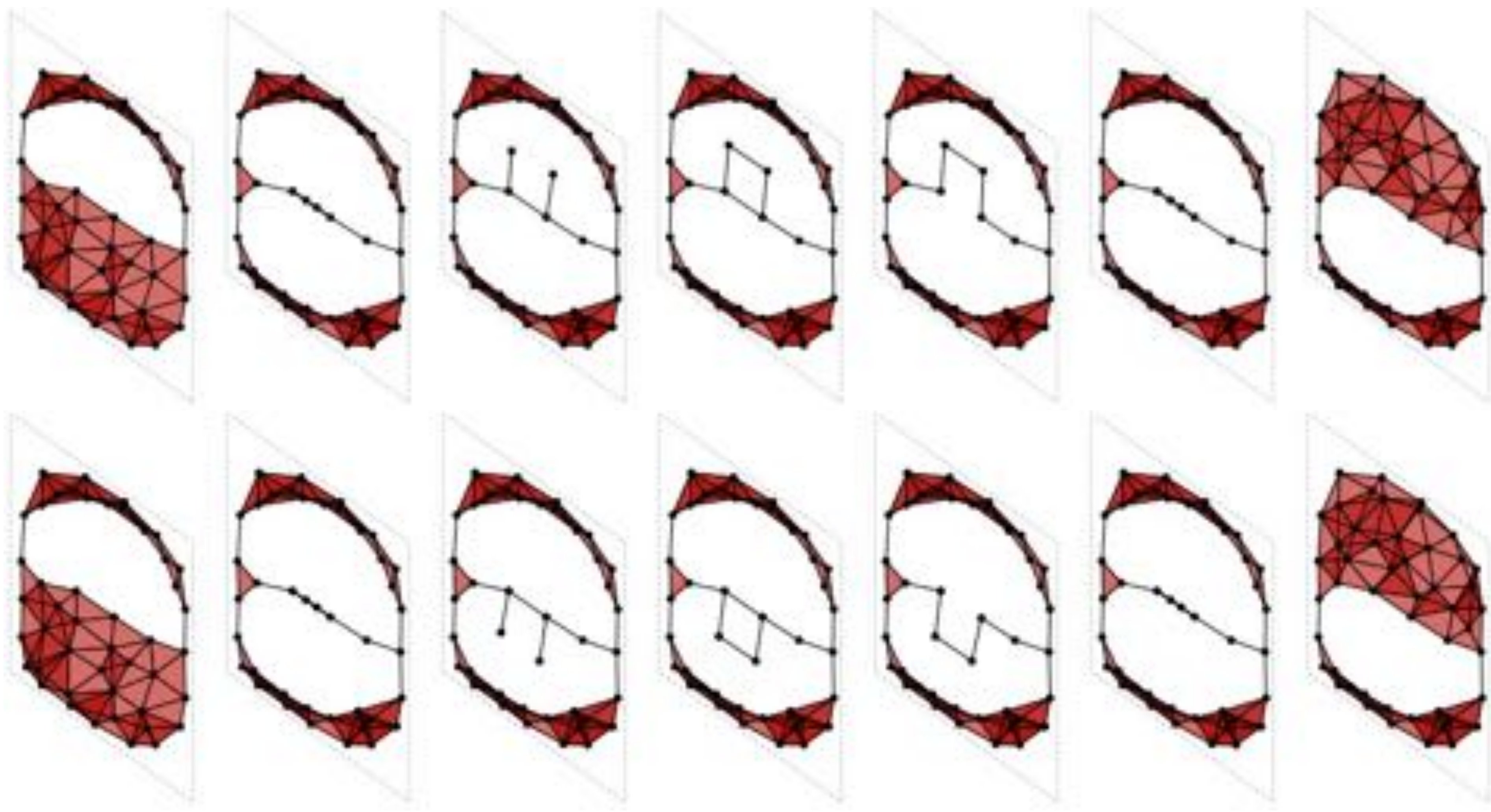

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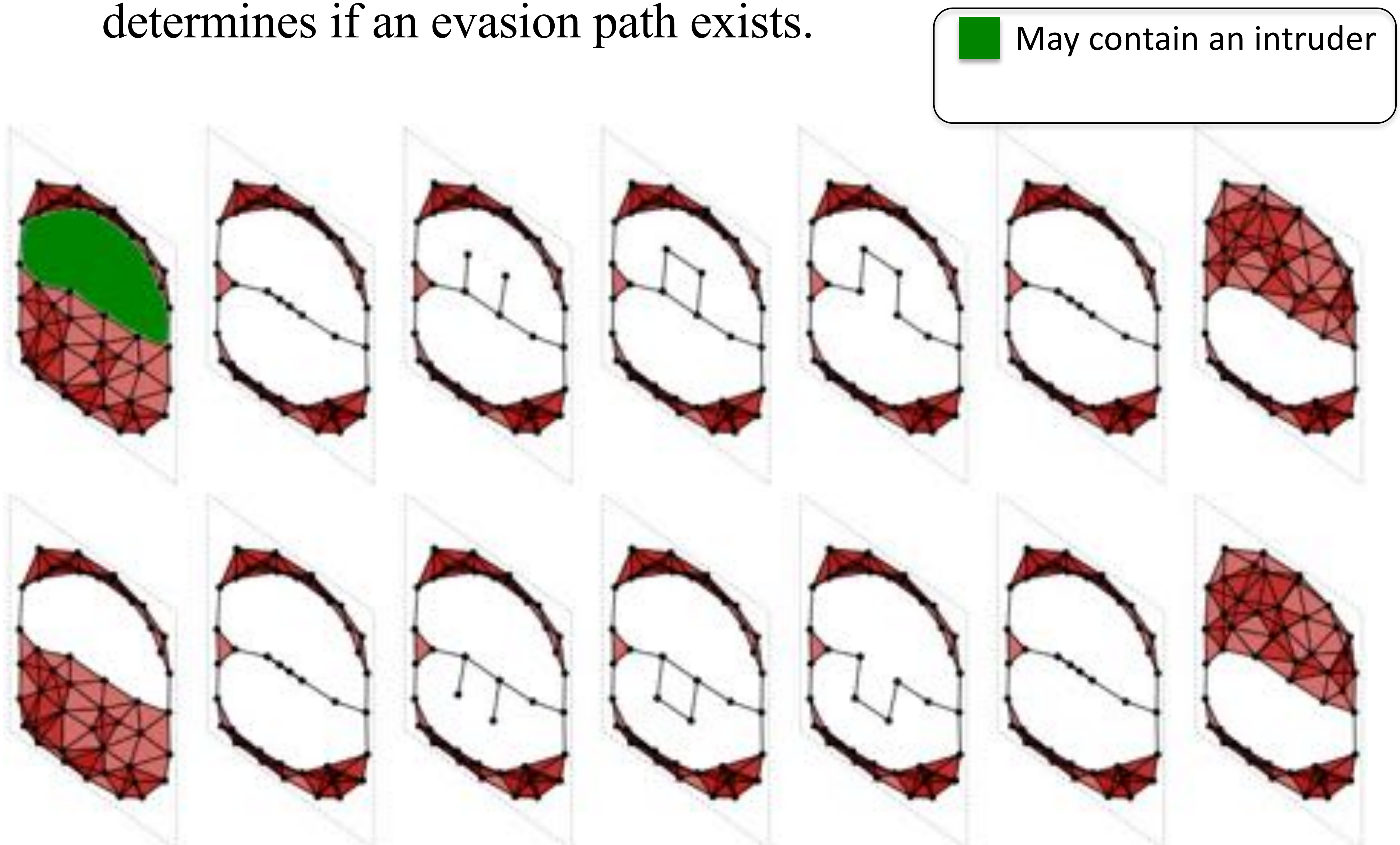

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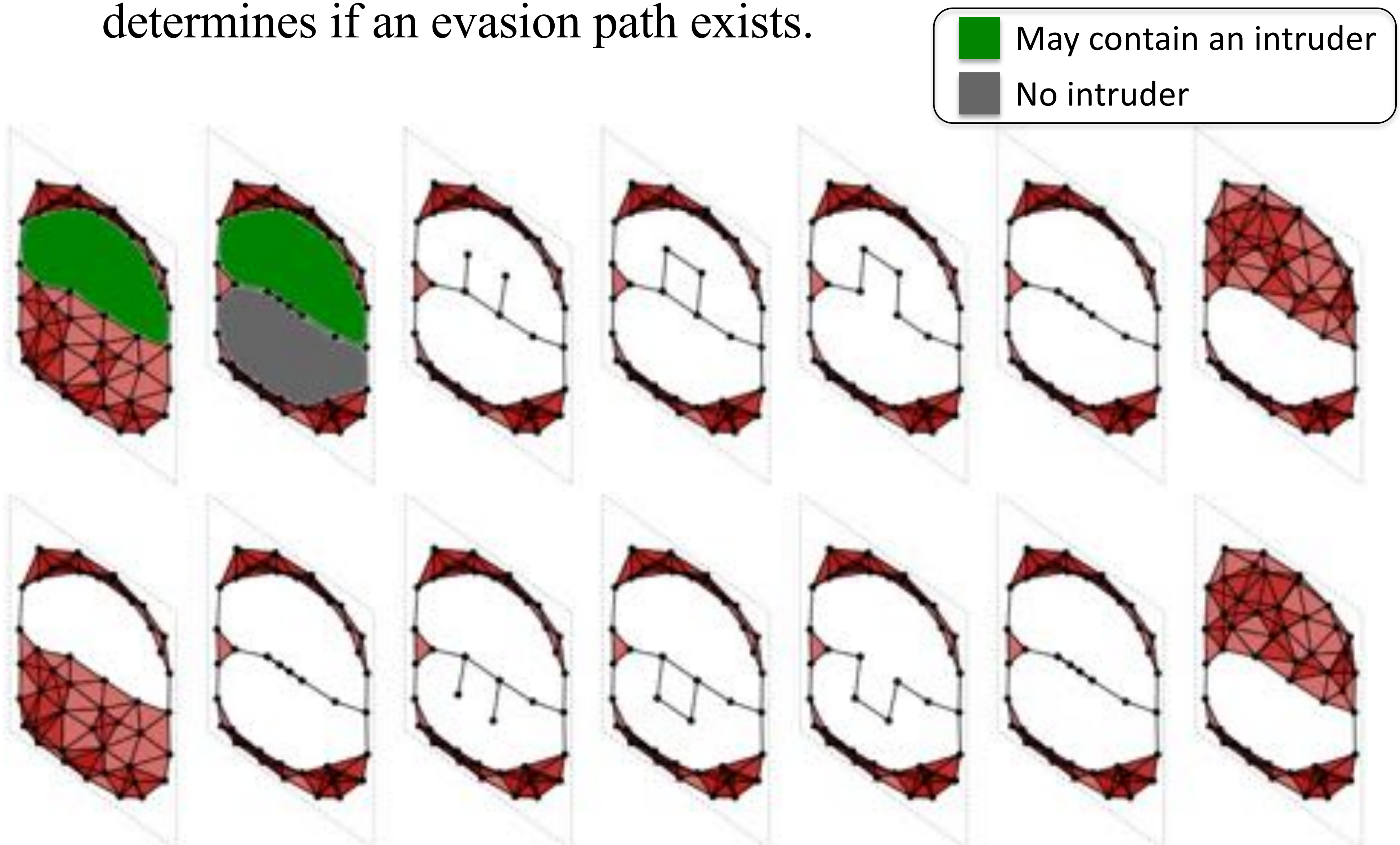

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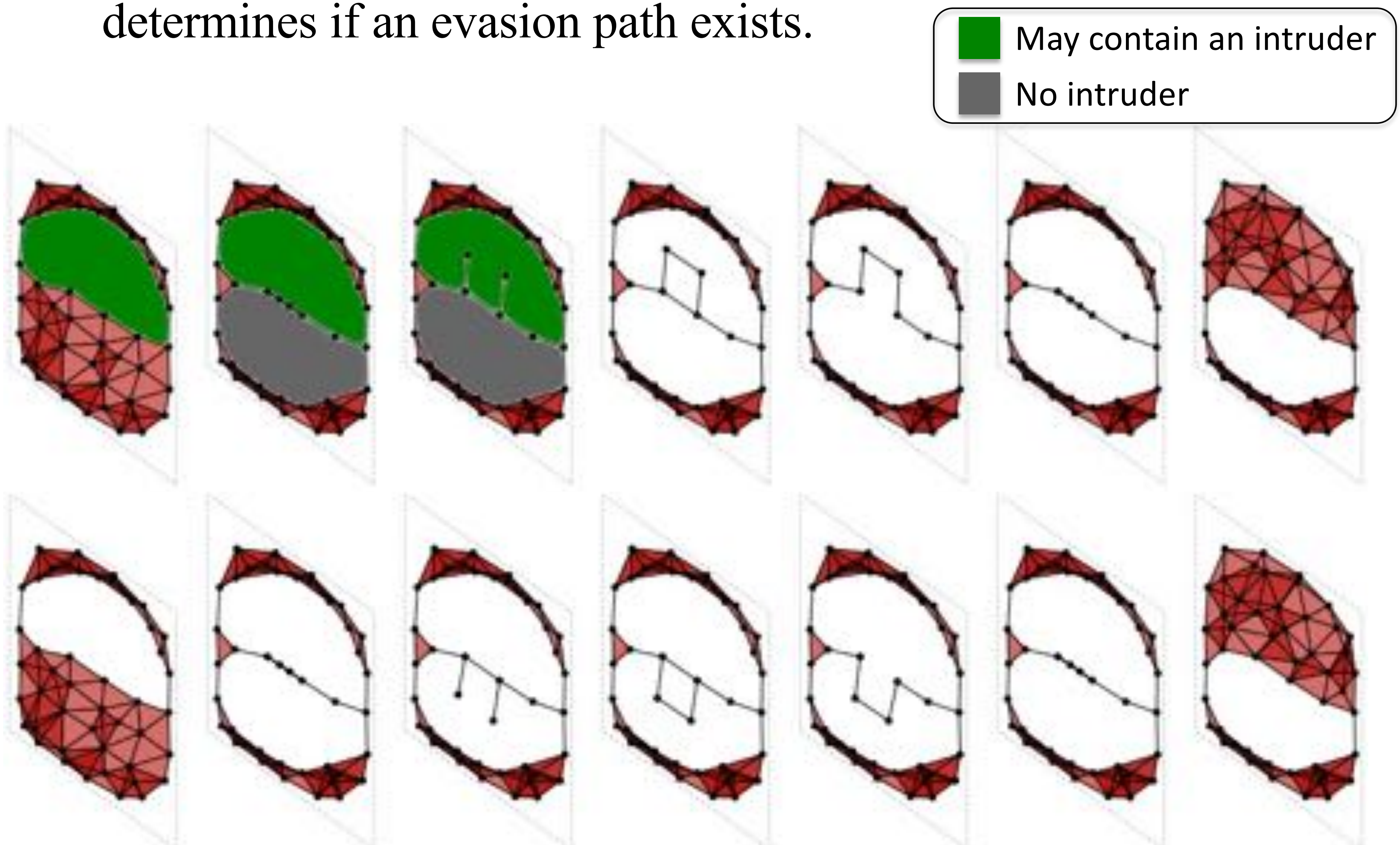

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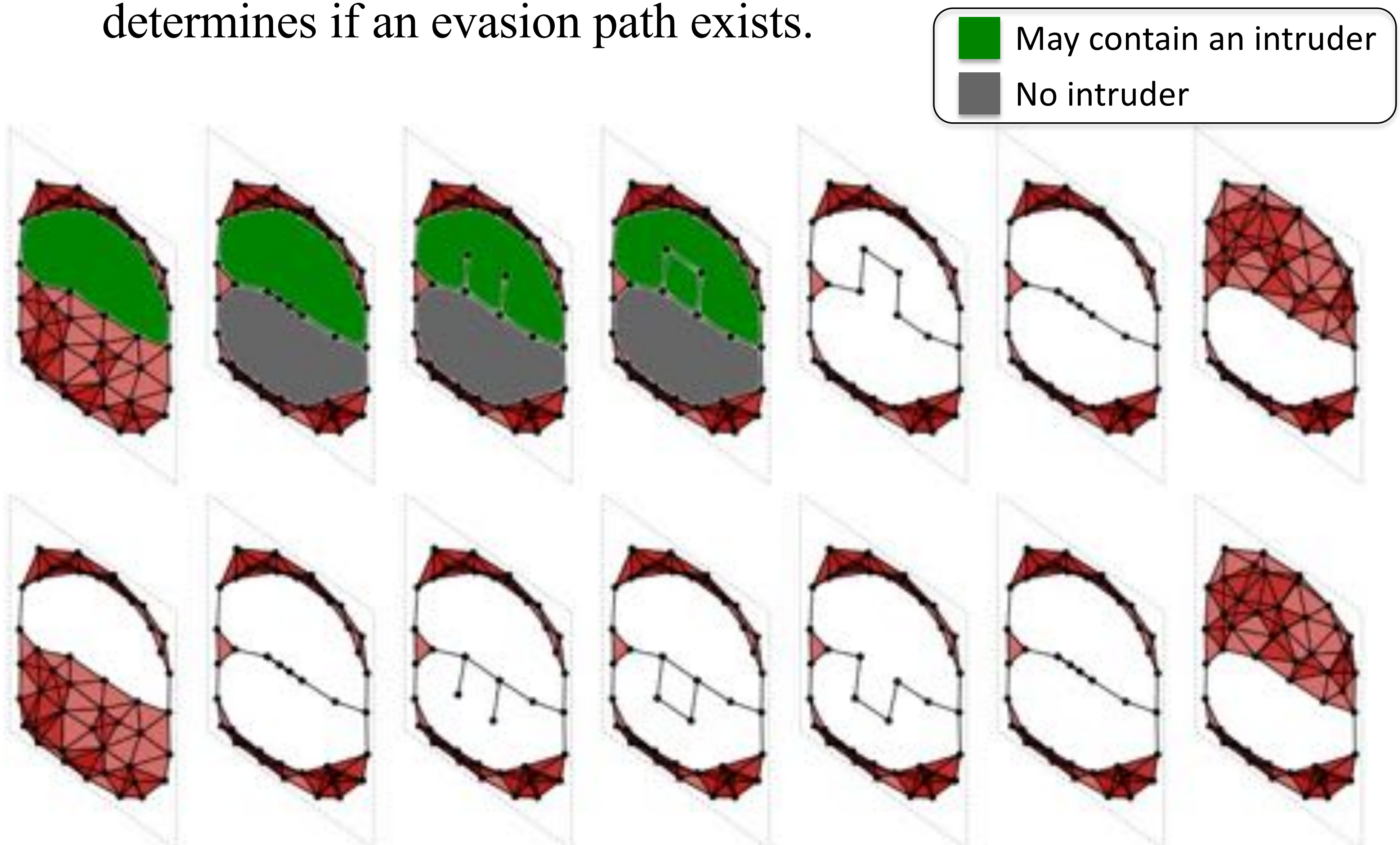

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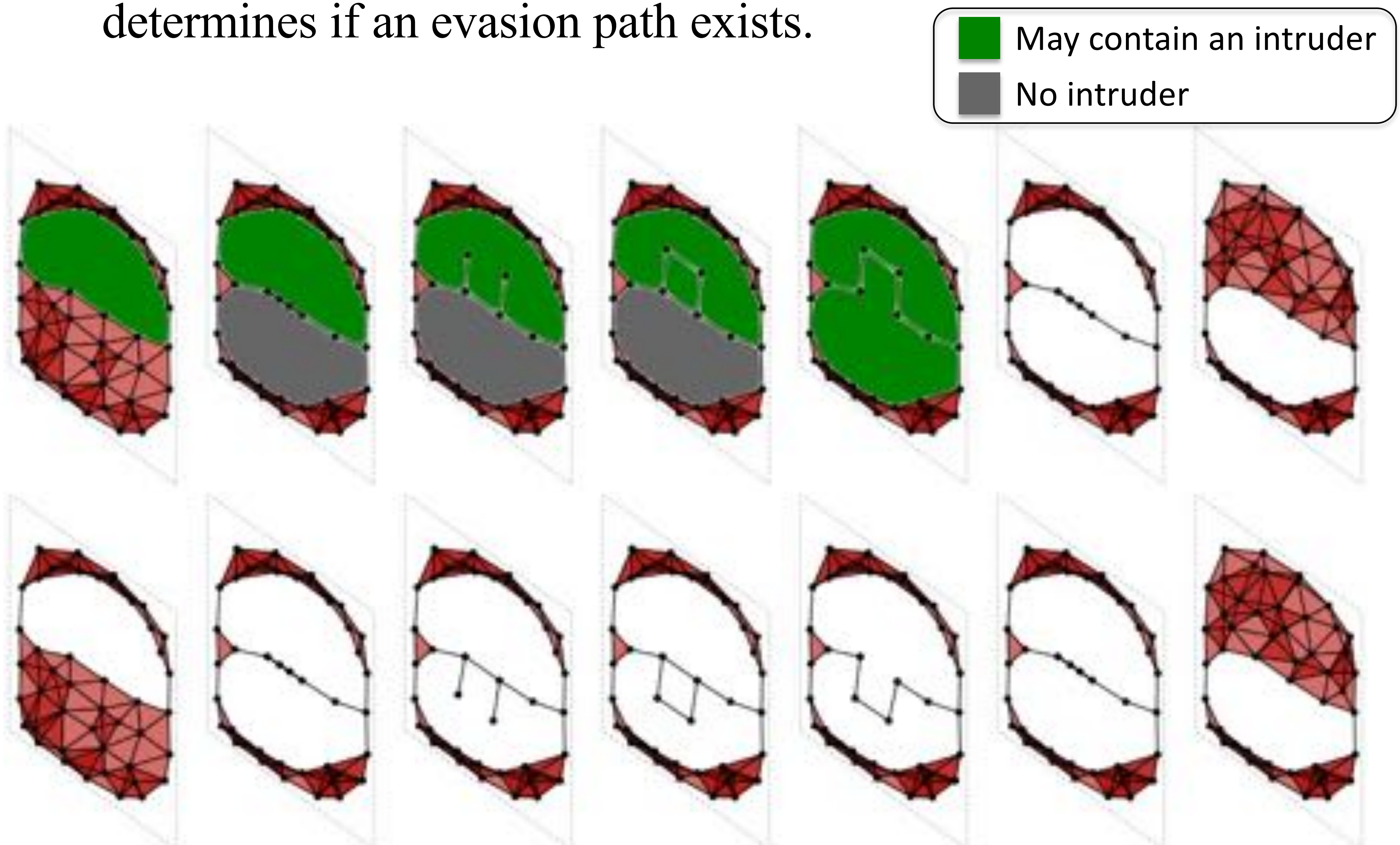

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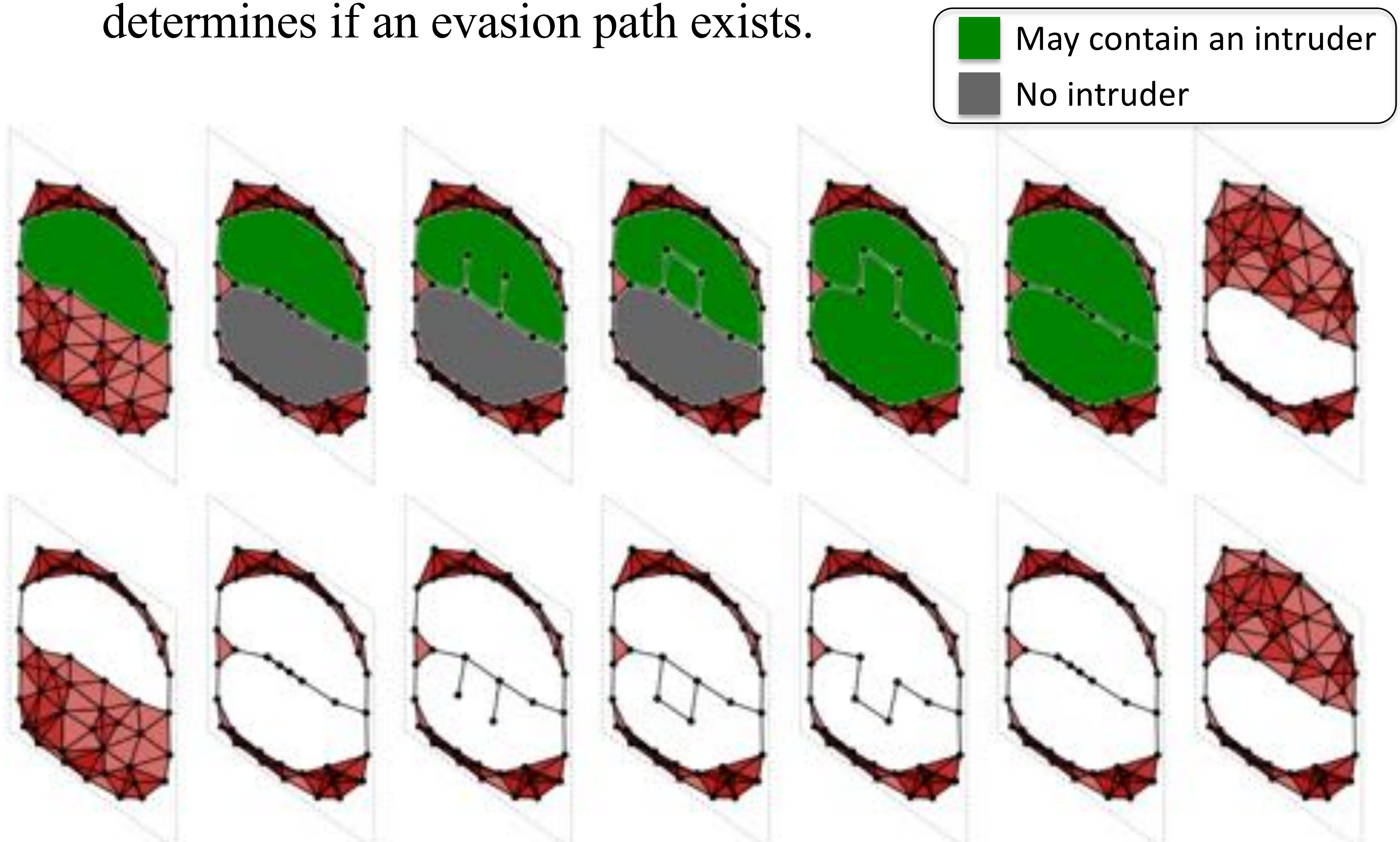

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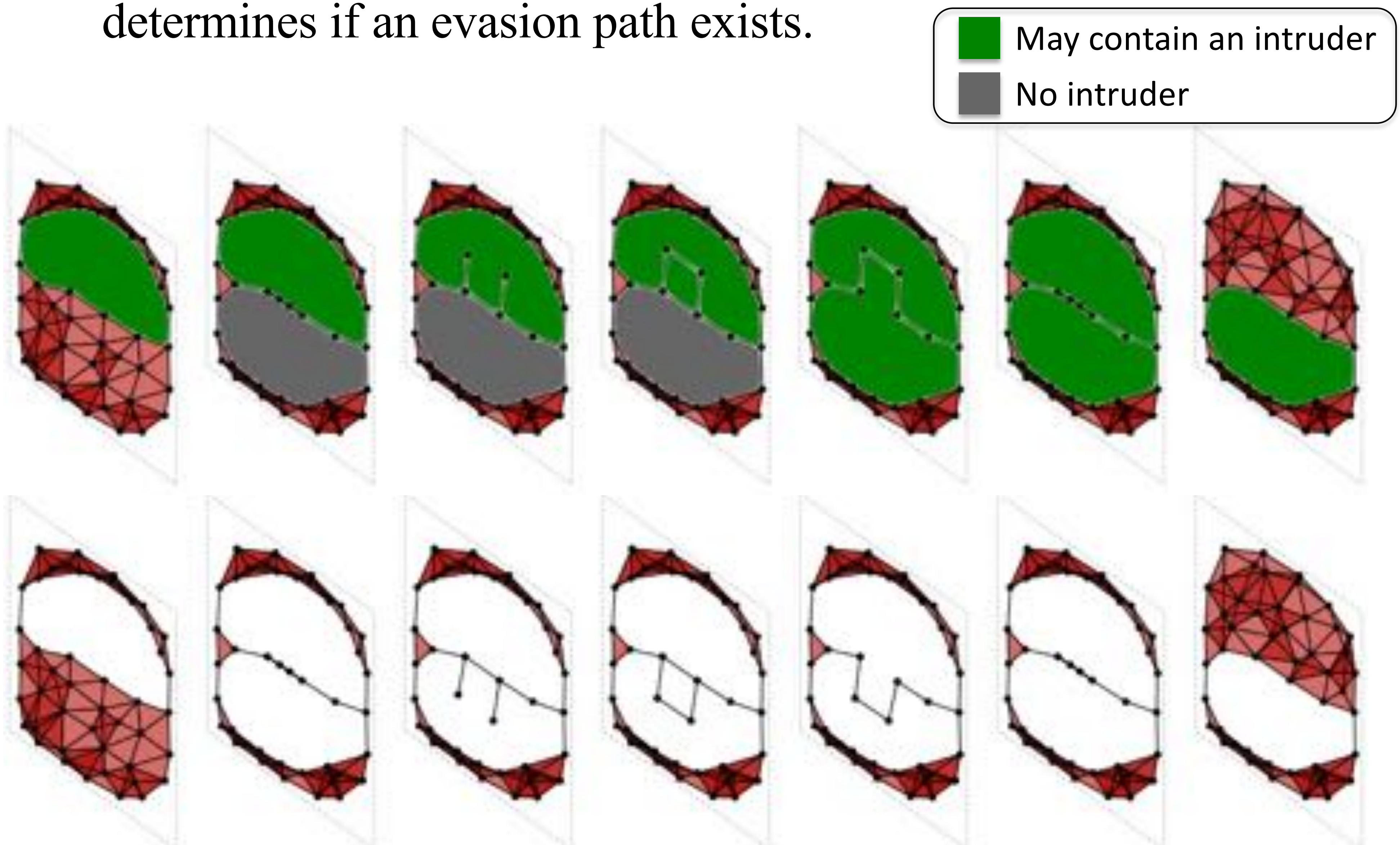

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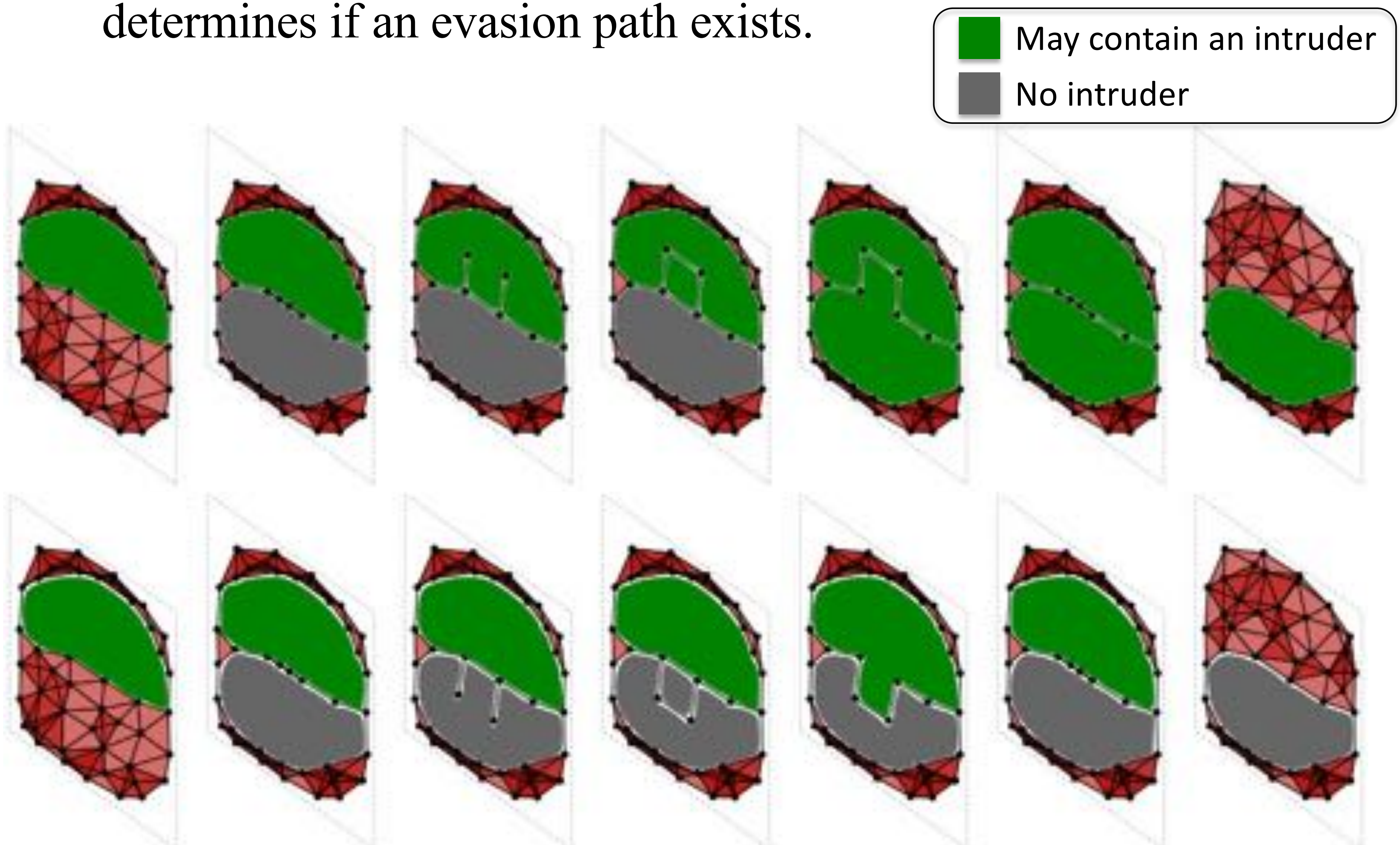

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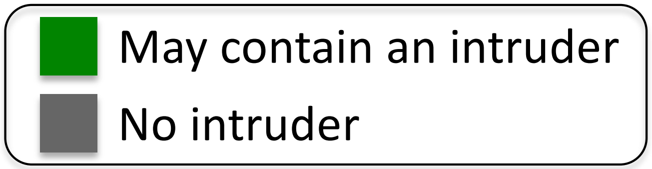
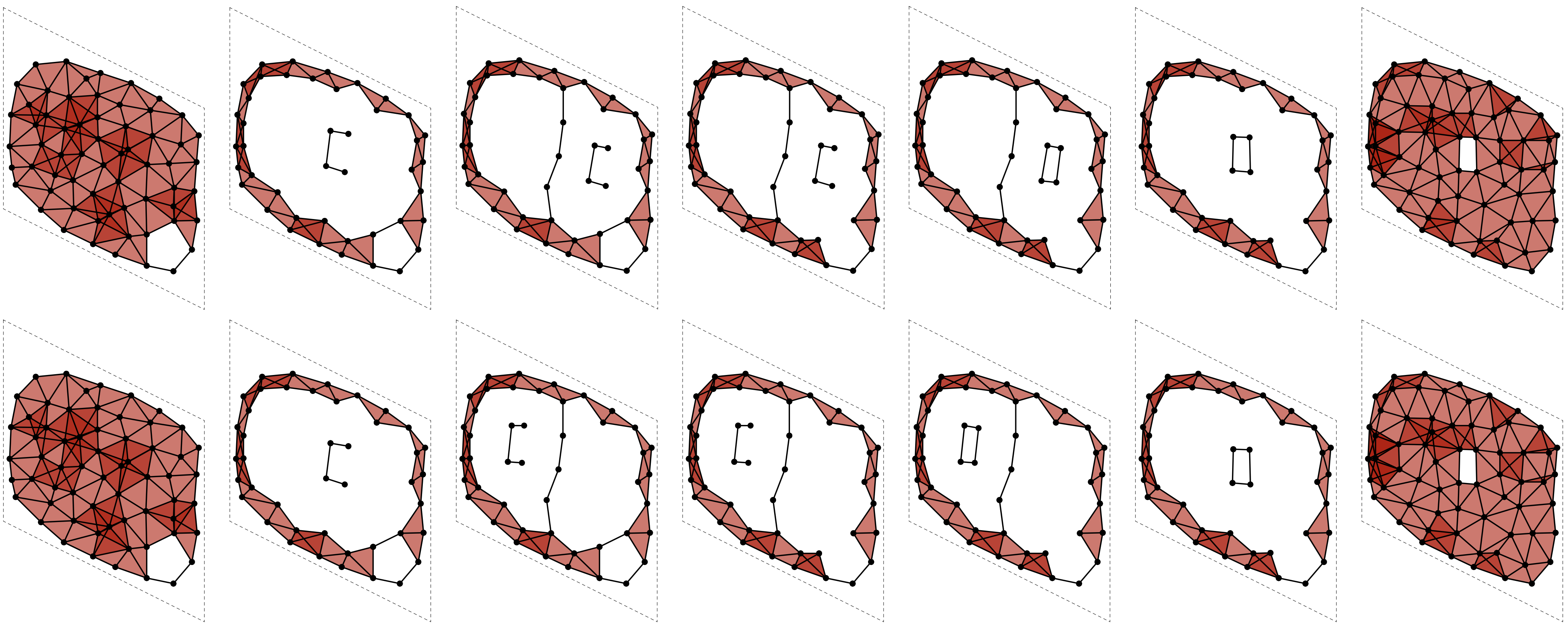
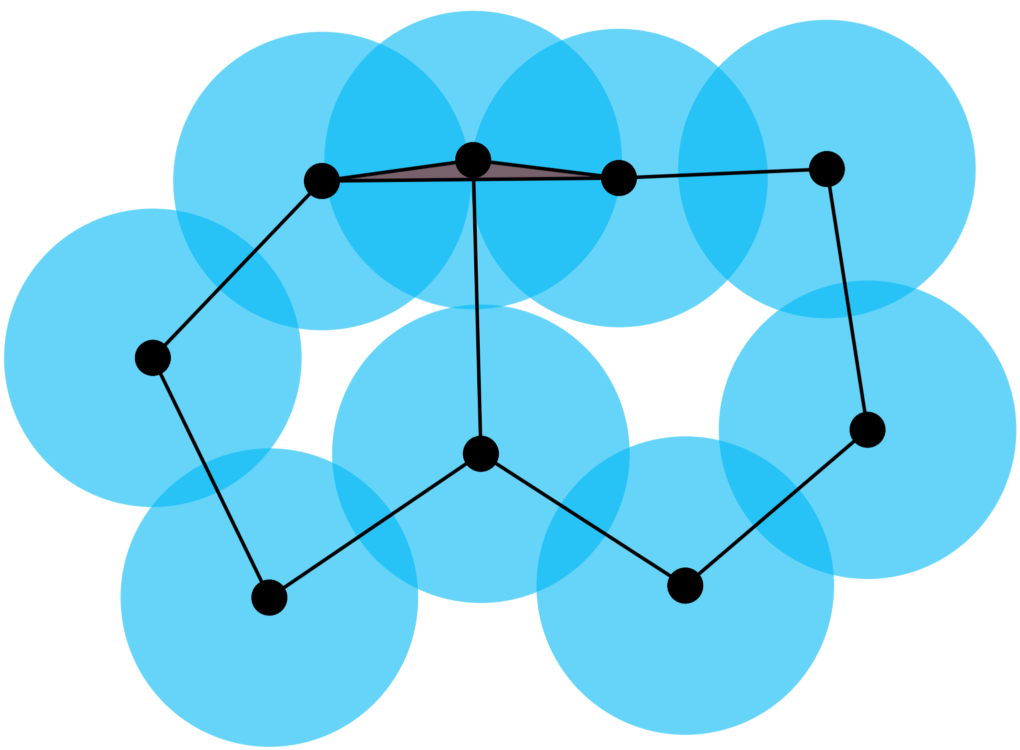




Planar sensors measuring cyclic orders

- <u>Theorem.</u> In a planar sensor network that remains connected, the time-varying alpha complex with rotation information determines if an evasion path exists.
- <u>Open question.</u> Is the Čech complex with rotation information sufficient?



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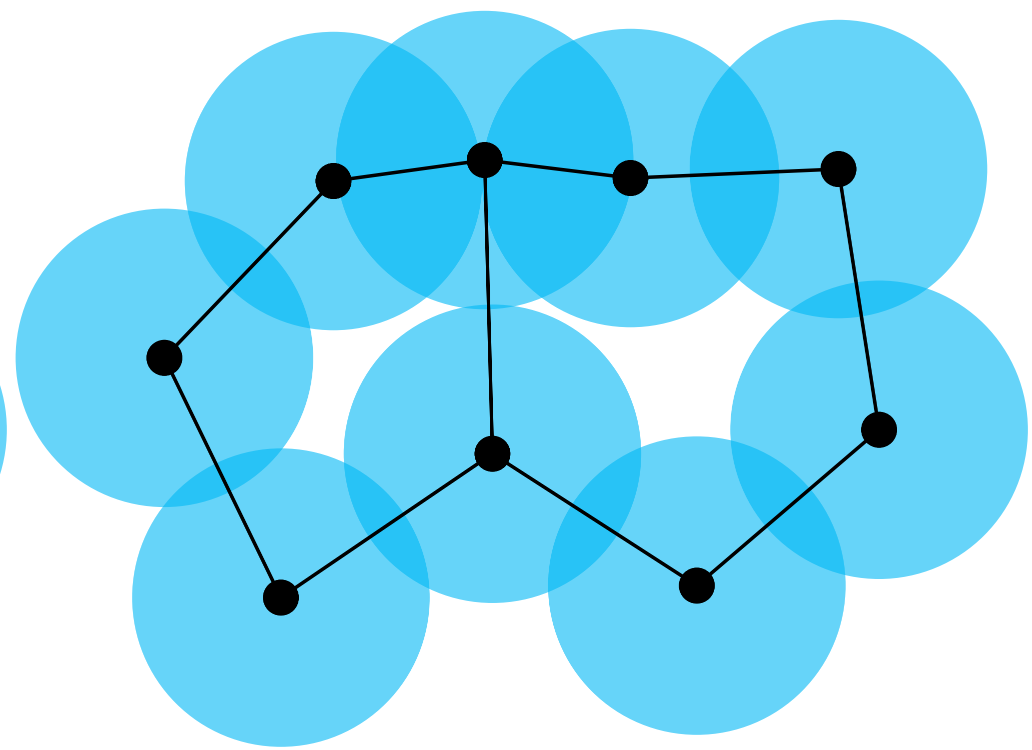

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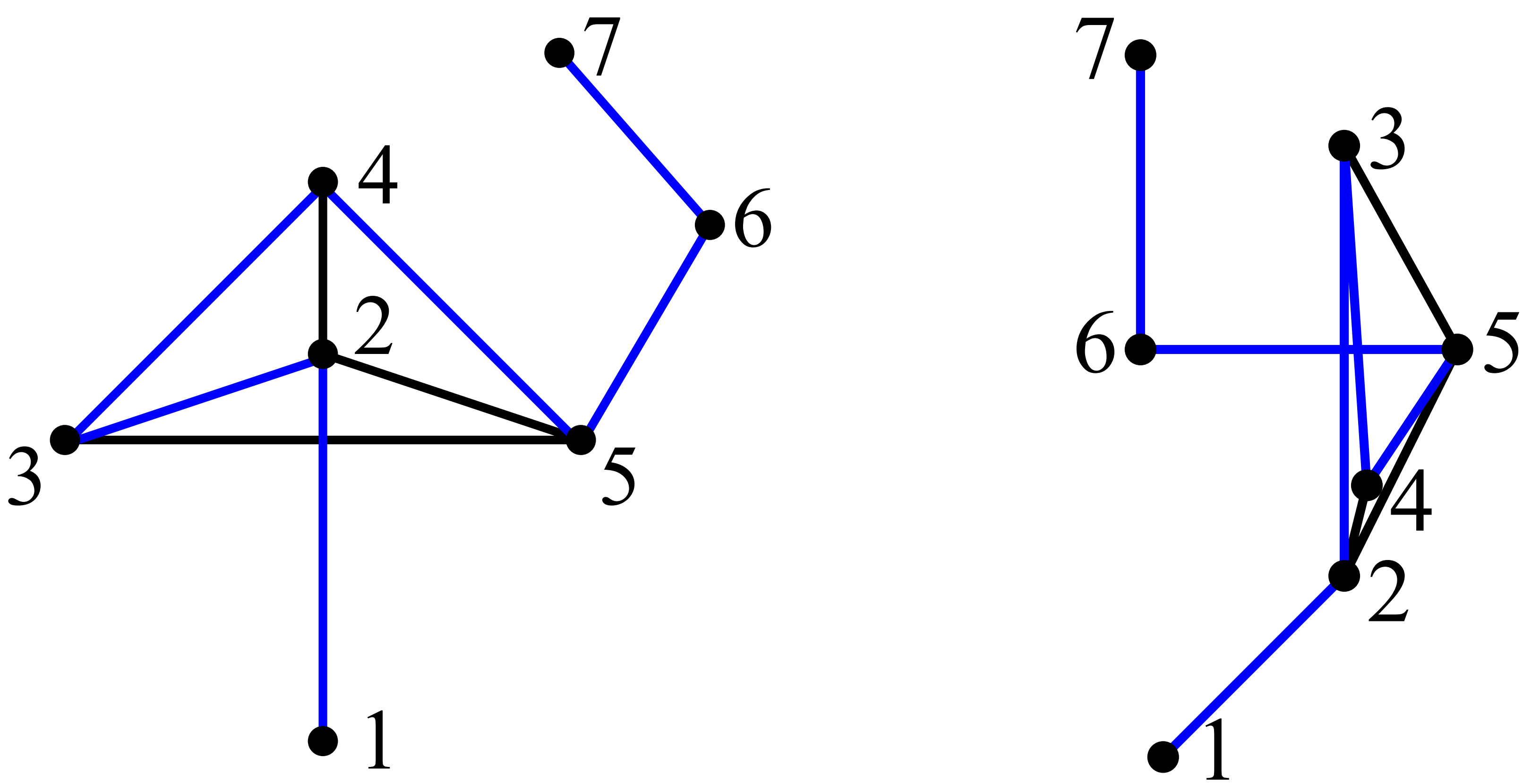

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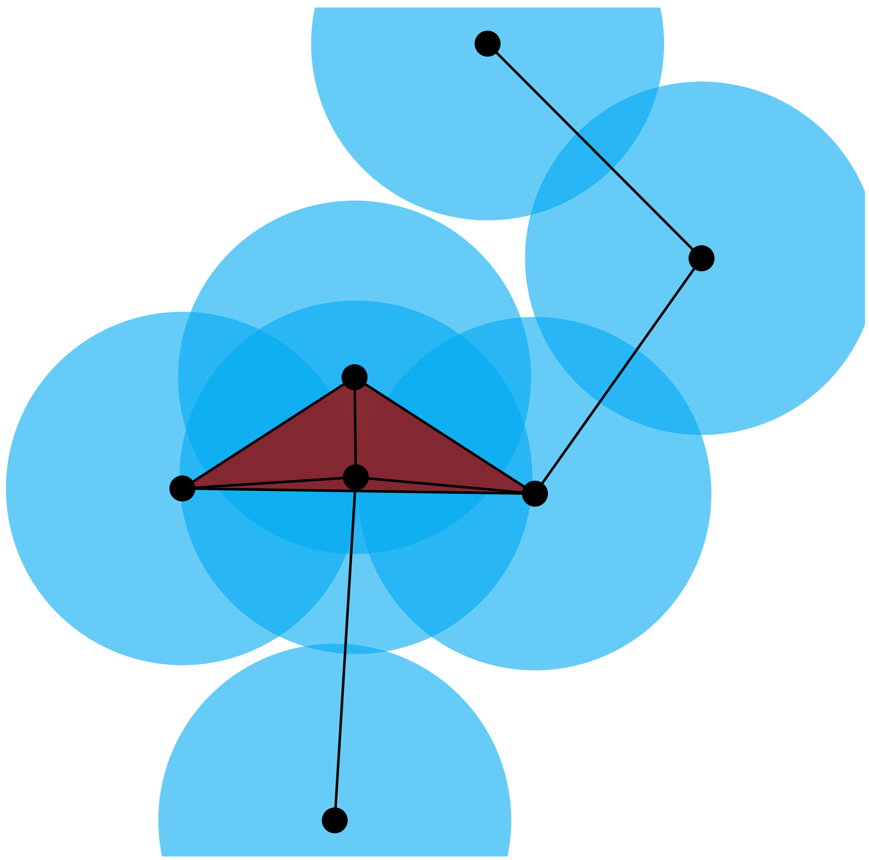

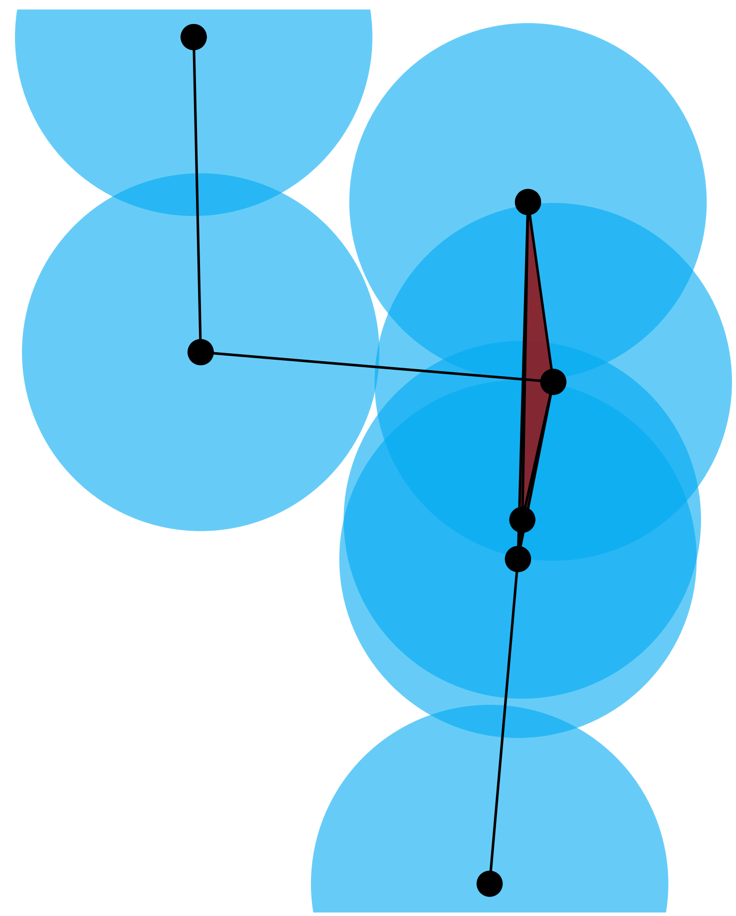

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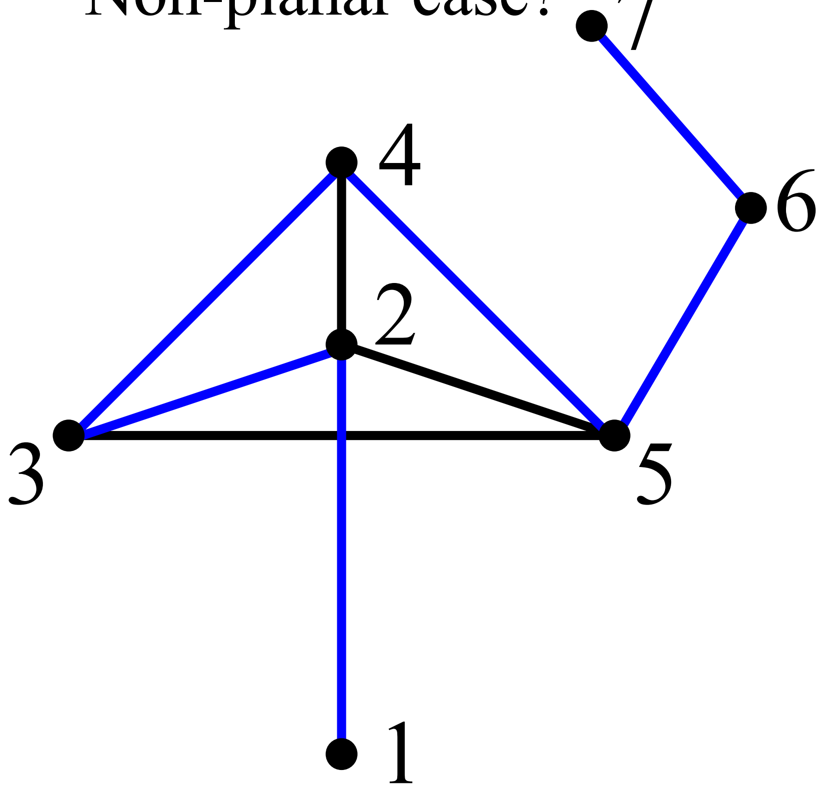
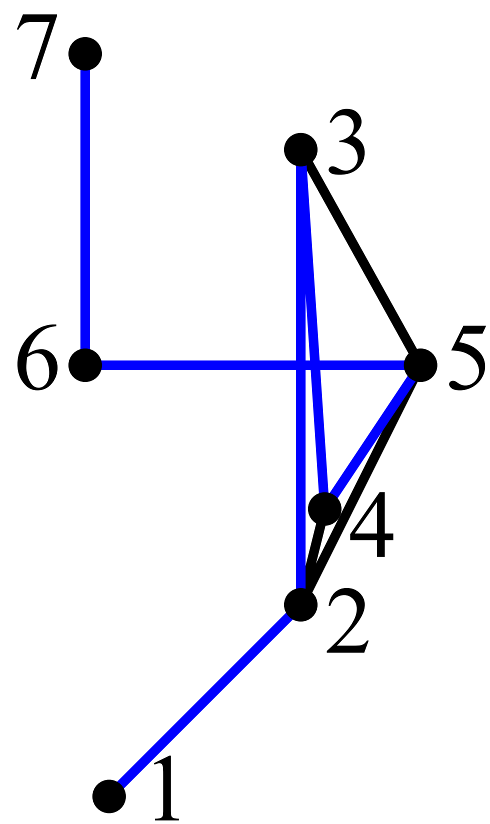
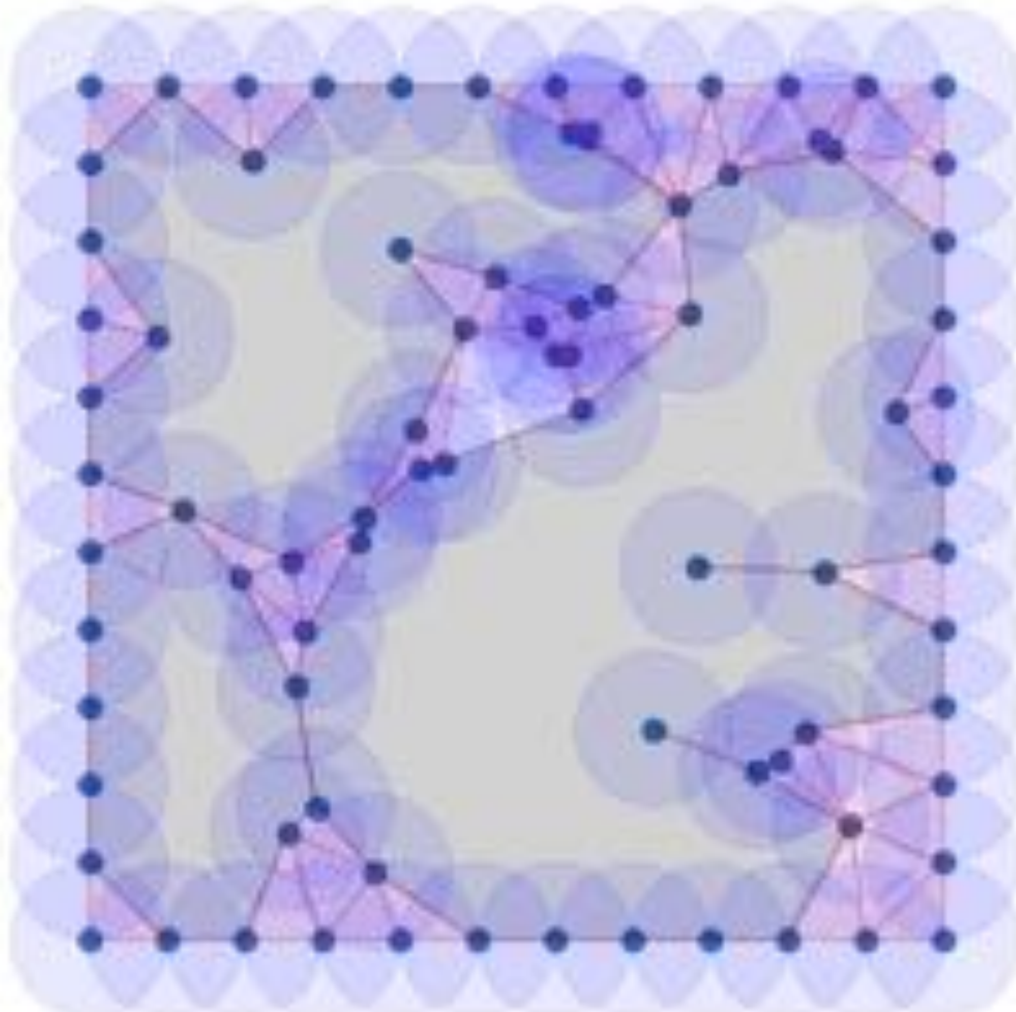
- Non-planar case?





Planar sensors measuring cyclic orders

- Expected time until mobile coverage for Brownian, billiard, and collective motion models.



Efficient Evader Detection in Mobile Sensor Networks by H. Adams, D. Ghosh, C. Mask, W. Ott, and K. Williams. https://github.com/elykwilliams/EvasionPaths

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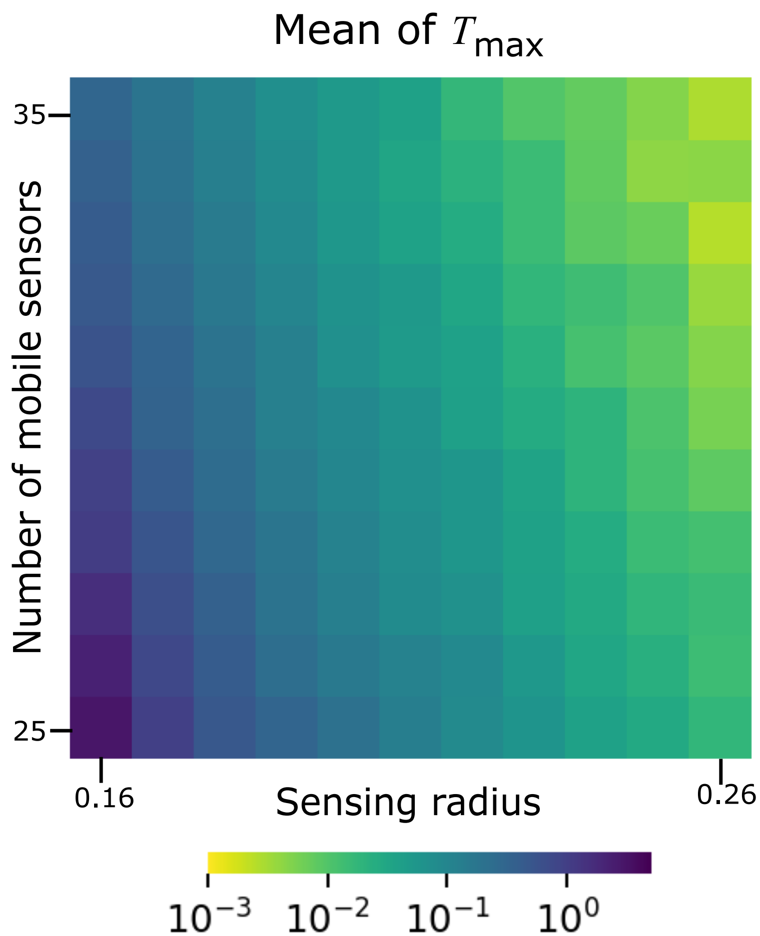

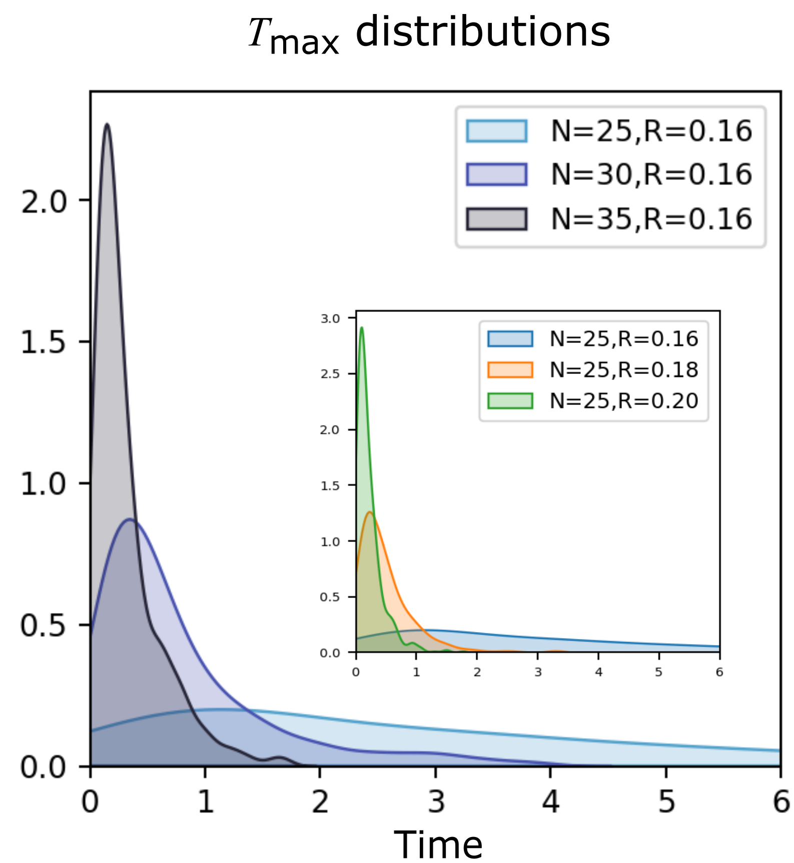

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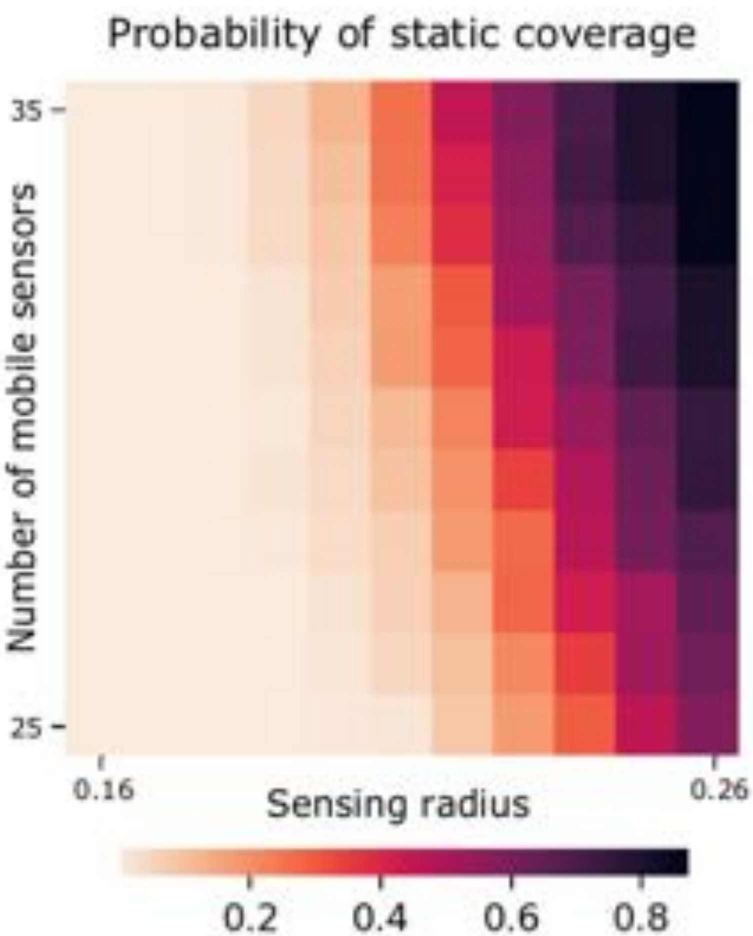

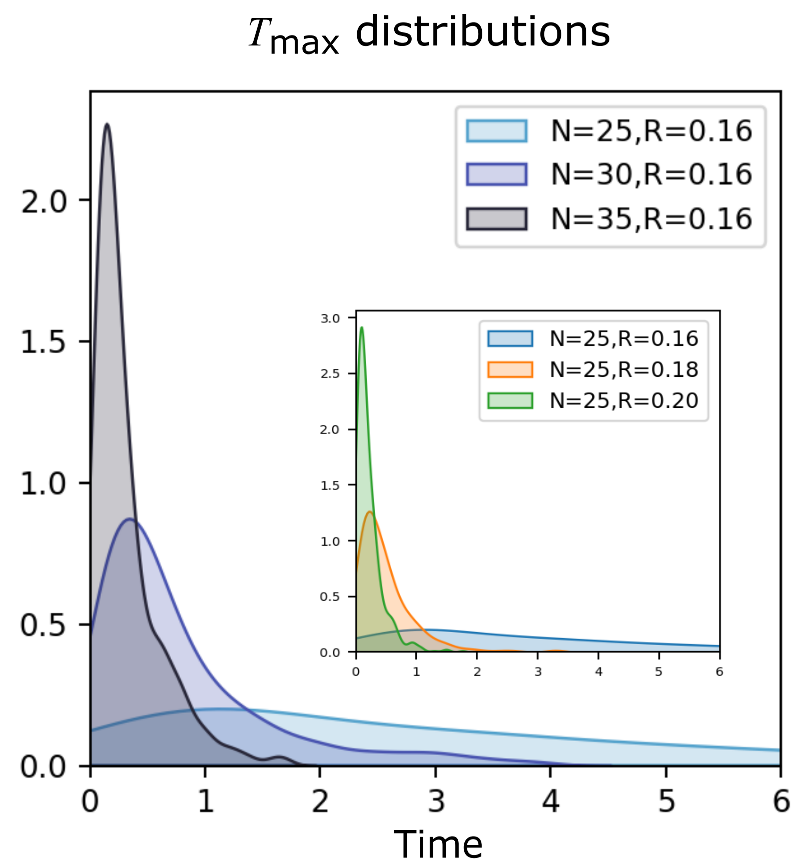

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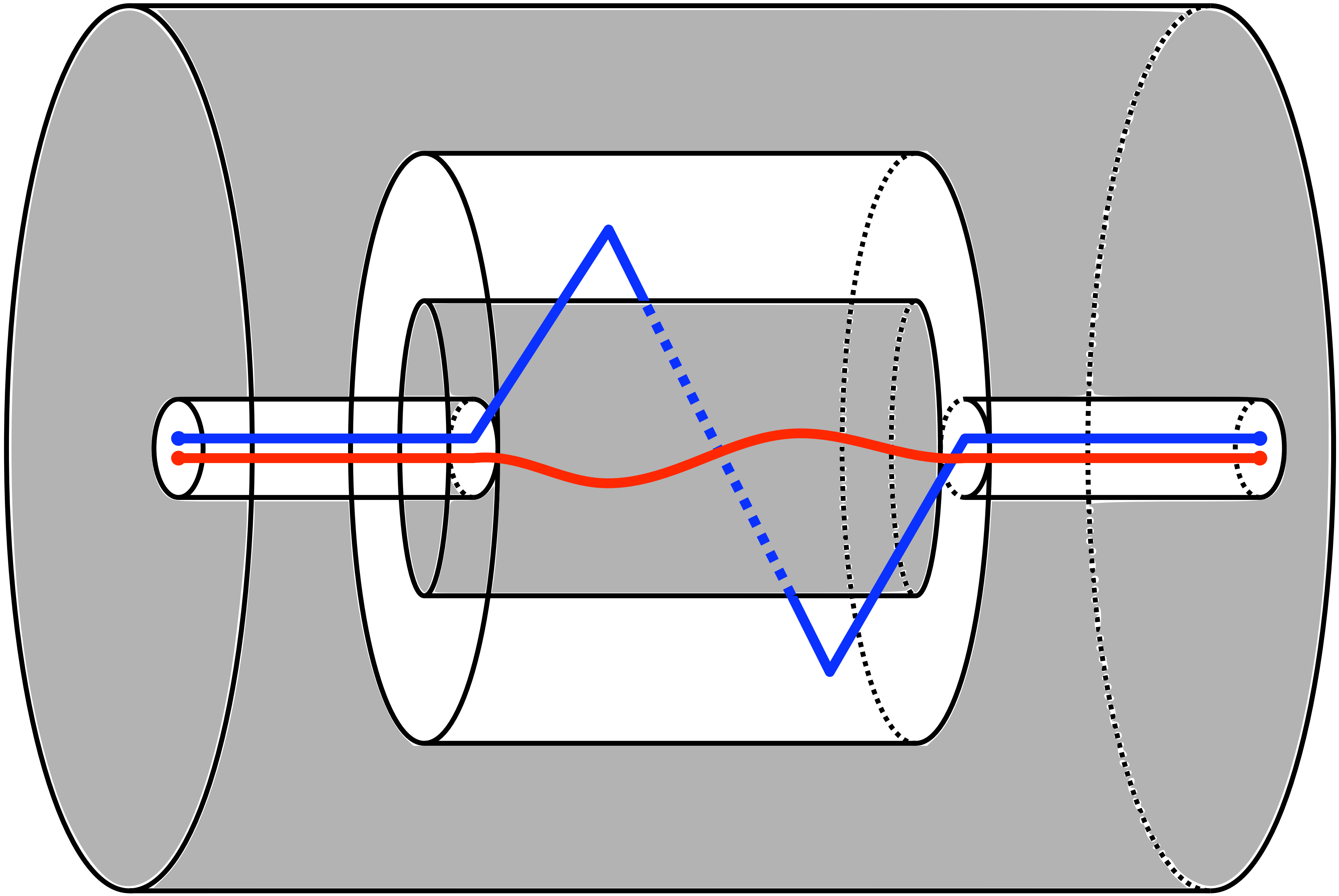
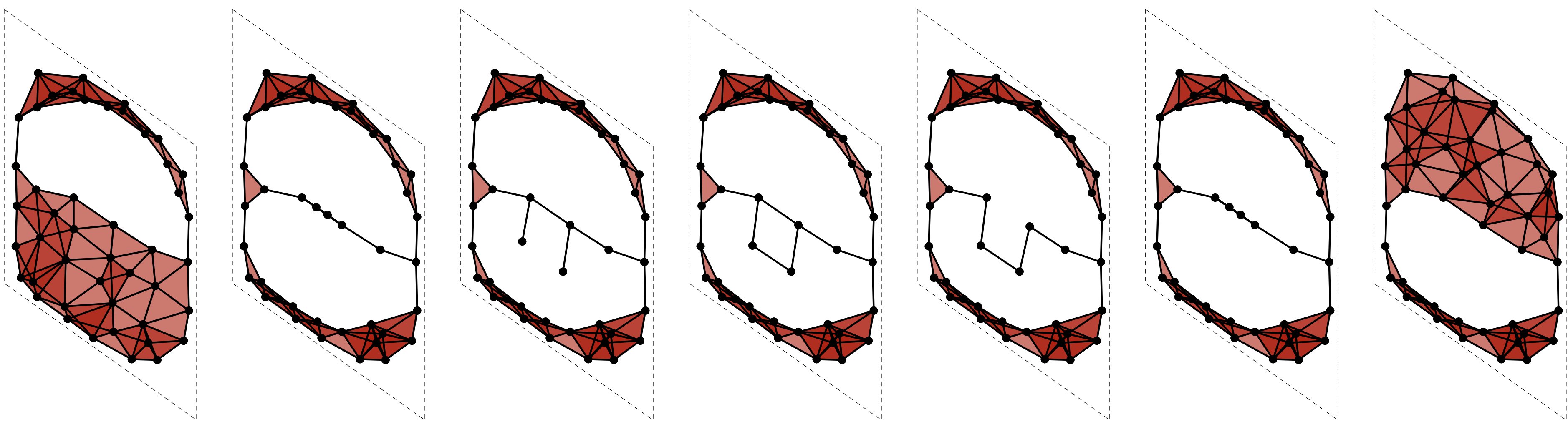
What’s the space of evasion paths?



Conclusions

- Streaming one-sided criterion using zigzag persistence.
- Cech complex insufficient.
  Alpha complex with rotation information suffices.
  What about the Čech complex with rotation information?



Vin de Silva and Robert Ghrist, *Coordinate-free coverage in sensor networks with controlled boundaries via homology*, International Journal of Robotics Research 25 (2006), 1205-1222.

Henry Adams and Gunnar Carlsson, *Evasion paths in mobile sensor networks*, International Journal of Robotics Research 34 (2015), 90-104.