UF MTG 4302/5316 **Exam 1**

Name: _____

- Explain your work (efficiently); partial credit is available.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:
  "I will not give, receive, or use any unauthorized assistance."

Signature: _____

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

UF MTG 4302/5316 **Exam 1**

1 Let A, X, and Y be topological spaces, and let $X \times Y$ have the product topology. Define $f\colon A \to X \times Y$ by $f(a) = (f_1(a), f_2(a))$, where $f_1\colon A \to X$ and $f_2\colon A \to Y$. Prove that if f_1 and f_2 are continuous, then f is continuous.

UF MTG 4302/5316 **Exam 1**

2 Let X be set. Prove that if $\mathcal{S}$ is a collection of subsets of X whose union equals X, then

$$\mathcal{B} = \{S_1 \cap \ldots \cap S_n \mid n \in \mathbb{Z}_+ \text{ and } S_i \in \mathcal{S} \text{ for all } i = 1, \ldots, n\}$$

satisfies the axioms so that $\mathcal{B}$ is a basis for a topology on X.

UF MTG 4302/5316 **Exam 1**

3 Let X be a topological space, and let $\{A_\alpha\}_{\alpha\in J}$ be a collection of subsets of X. Prove that $\cup_{\alpha\in J}\overline{A_\alpha} \subset \overline{\cup_{\alpha\in J}A_\alpha}$. Give an example where the two sets are not equal, i.e. where we have strict containment.

Recall that $\overline{A}$ denotes the closure of a set A in X.

4 Let A be a partially ordered set. The maximum principal states that there is a maximal totally ordered subset B of A. Zorn's lemma states that if every totally ordered subset of A has an upper bound in A, then A has a maximal element. Prove that the maximum principal implies Zorn's lemma.

UF MTG 4302/5316 **Exam 1**

5 (a) Prove that a countable union of countable sets is countable.

(b) Prove that a countable product of finite sets (for example $\{0,1\}^\omega$) need not be countable.

This page intentionally left blank.

This page intentionally left blank.

This page intentionally left blank.