UF MTG 6346 **Exam 1**

Name: ____

- Explain your work (efficiently); partial credit is available.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:
  "I will not give, receive, or use any unauthorized assistance."

Signature: ____

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

1. Let X and Y be topological spaces with basepoints x_0 and y_0. Suppose $\varphi\colon (X, x_0) \to (Y, y_0)$ is continuous. How is the induced map $\varphi_*\colon \pi_1(X, x_0) \to \pi_1(Y, y_0)$ on fundamental groups defined? Why is this well-defined? Verify that φ_* is a group homomorphism.

2 Let a and b be the generators of $\pi_1(S^1 \vee S^1)$ corresponding to the two S^1 summands. Draw a picture of the covering space of $S^1 \vee S^1$ corresponding to the normal subgroup generated by a^2, b^2, and $(ab)^4$, and prove that this covering space is indeed the correct one.

3 Let (X, x_0), $(\tilde{X}_1, \tilde{x}_1)$, and $(\tilde{X}_2, \tilde{x}_2)$ be path-connected and locally path-connected spaces with distinguished basepoints. Let $p_1\colon (\tilde{X}_1, \tilde{x}_1) \to (X, x_0)$ and $p_2\colon (\tilde{X}_2, \tilde{x}_2) \to (X, x_0)$ be basepoint-preserving covering space maps.

- (3 points) Define what it means for $f\colon (\tilde{X}_1, \tilde{x}_1) \to (\tilde{X}_2, \tilde{x}_2)$ to be an *isomorphism of covering spaces.*
- (7 points) Prove that the covering spaces p_1 and p_2 are isomorphic if and only if $p_{1*}(\pi_1(\tilde{X}_1, \tilde{x}_1)) = p_{2*}(\pi_1(\tilde{X}_2, \tilde{x}_2))$.

4 Recall the wedge sum $\vee_\alpha X_\alpha$ of a collection of spaces X_α with basepoints $x_\alpha \in X_\alpha$ is the quotient space of the disjoint union $\coprod_\alpha X_\alpha$ in which all the basepoints $x_\alpha \in X_\alpha$ are identified to a single point. Suppose each X_α is path-connected, and suppose each x_α is a deformation retract of an open neighborhood U_α in X_α. Use van Kampen's Theorem to prove that $\pi_1(\vee_\alpha X_\alpha) \cong *_\alpha \pi_1(X_\alpha)$.

5 Just say "True" or "False". No justification is required; no partial credit is available.

(a) There is a retract from the torus $S^1 \times S^1$ to the circle $S^1 \times \{y_0\}$, where $y_0 \in S^1$.

(b) For all $n \geq 0$, the fundamental group $\pi_1(\mathbb{C}P^n)$ of complex projective space $\mathbb{C}P^n$ is trivial.

(c) Let S^2 be the 2-sphere and let $\mathbb{R}P^2$ be the projective plane. There exists a covering space map $p\colon S^2 \to \mathbb{R}P^2$ which furthermore is the universal cover of $\mathbb{R}P^2$.

(d) If $p\colon (\tilde{X}, \tilde{x}_0) \to (X, x_0)$ is a normal covering space, then $G(\tilde{X}) \cong p_*(\pi_1(\tilde{X}, \tilde{x}_0))$.

(e) If X is a CW complex and Y is a 0-dimensional CW complex, then their join $X * Y$ is contractible.

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