UF MTG 6347 **Exam 1**

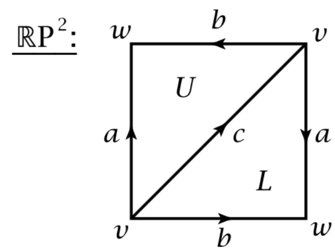
Name: ________________________________

- Explain your work (efficiently); partial credit is available.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:
  "I will not give, receive, or use any unauthorized assistance."

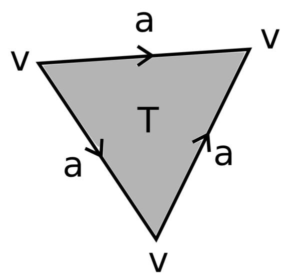
Signature: ____

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

1 Directly from the definitions, compute the simplicial cohomology groups of $\mathbb{R}P^2$ with $\mathbb{Z}_2 := \mathbb{Z}/2\mathbb{Z}$ coefficients, using the Δ-complex structure given below.



2 Compute the simplicial cohomology groups of the Δ-complex X below, with $\mathbb{Z}$ coefficients. (Note the orientation of the three edges carefully.)



3 Let X be a space and let R be a ring.

(a) Define the cup product $\cup\colon C^k(X;R) \times C^l(X;R) \to C^{k+l}(X;R)$ at the level of singular cochains.

(b) Recall that for $\phi \in C^k(X;R)$ and $\psi \in C^l(X;R)$, we have

$$\delta(\phi \cup \psi) = (\delta\phi \cup \psi) + (-1)^k(\phi \cup \delta\psi).$$

Prove the cup product of a cocycle and a coboundary is a coboundary, and similarly prove the cup product of a coboundary and a cocycle is a coboundary.

4 Let $f\colon X \to Y$ be a continuous map between two topological spaces X and Y. Let $n \geq 0$.

(a) (3 points) Use f to define a map $f^{\#}\colon C^n(Y;\mathbb{Z}) \to C^n(X;\mathbb{Z})$ from the n-cochains on Y to the n-cochains on X (note this map is contravariant).

(b) (4 points) Verify the map in (a) satisfies $\delta f^{\#} = f^{\#}\delta$.

(c) (3 points) Deduce that $f^{\#}$ induces a map $f^*\colon H^n(Y;\mathbb{Z}) \to H^n(X;\mathbb{Z})$.

5 Just say “True” or “False”. No justification is required; no partial credit is available.

(a) Let K and L be simplicial complexes. If $f, g\colon K \to L$ are two simplicial maps that agree on the vertices of K, then $f = g$.

(b) If the spaces X and Y have isomorphic cohomology rings, meaning there is a ring isomorphism $H^*(X;\mathbb{Z}) \cong H^*(Y;\mathbb{Z})$, then X and Y are homotopy equivalent.

(c) For n even, every map $f\colon \mathbb{R}P^n \to \mathbb{R}P^n$ has a fixed point.

(d) If M is an orientable surface, then $\alpha \cup \alpha = 0$ for any $\alpha \in H^1(M;\mathbb{Z})$.

(e) If G is an abelian group and A is a free abelian group, then $\operatorname{Ext}(A, G) = 0$.

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