UF MTG 4302/5316 **Exam 2**

Name: ____

- Explain your work (efficiently); partial credit is available.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:
  "I will not give, receive, or use any unauthorized assistance."

Signature: ____

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

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$\boxed{1}$ Define what it means for a topological space X to be connected.

Let $f\colon X \to Y$ be a continuous map between topological spaces. Show that if X is connected, then $f(X)$ is connected.

$\boxed{2}$ Define what it means for a topological space X to be compact. If X is compact and C is closed in X, then prove that C is compact.

$\boxed{3}$ Let $\mathbf{x}_1, \mathbf{x}_2, \ldots$ be a sequence of points in the product space $X = \prod_{\alpha \in J} X_\alpha$, and let $\mathbf{x} \in X$. For $\alpha \in J$, recall that $\pi_\alpha \colon X \to X_\alpha$ is the projection map. Show that if the sequence $\pi_\alpha(\mathbf{x}_1), \pi_\alpha(\mathbf{x}_2), \ldots$ converges to $\pi_\alpha(\mathbf{x})$ for each $\alpha \in J$, then the sequence $\mathbf{x}_1, \mathbf{x}_2, \ldots$ converges to $\mathbf{x}$.

$\boxed{4}$ Let X be compact, let Y be Hausdorff, and let $f\colon X \to Y$ be a continuous bijection. Prove that $f^{-1}\colon Y \to X$ is continuous.

Remark: It follows that f is a homeomorphism.

$\boxed{5}$ Let X be a topological space and let $A \subset X$. Show that if there is a sequence of points in A converging to x, then $x \in \overline{A}$. Show that the converse holds if X is metrizable.

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