UF MTG 6346 **Exam 2**

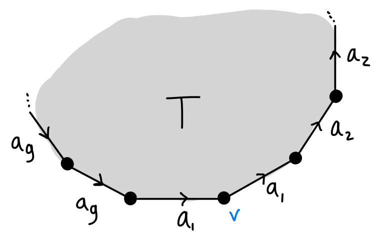
Name: ____

- Explain your work (efficiently); partial credit is available.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:
  "I will not give, receive, or use any unauthorized assistance."

Signature: ____

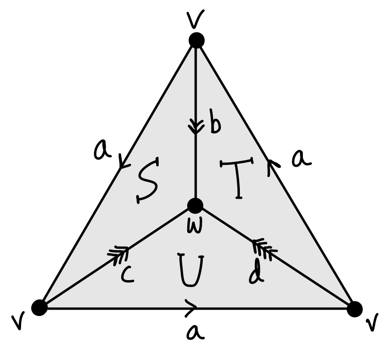
Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

1 Let X be the nonorientable surface of genus g, which has one 0-cell v, g different 1-cells $a_1, \ldots, a_g$, and one 2-cell attached by the word $a_1^2 a_2^2 \ldots a_g^2$. Compute the cellular homology $H_i(X)$ for $i = 0, 1, 2$.



2 Use a long exact sequence of your choosing to prove that $\tilde{H}_n(X) \cong \tilde{H}_{n+1}(SX)$ for all n, where SX is the suspension of space X.

3 Let X be the Δ-complex drawn below, which has two 0-simplices v and w, four 1-simplices a, b, c, d, and three 2-simplices S, T, U. Compute the simplicial homology $H_i(X)$ for $i = 0, 1, 2$.



4 (a) (7 points) Let $n \geq 1$, and let x be any point in $\mathbb{R}^n$. Use the long exact sequence of the pair of spaces $(\mathbb{R}^n, \mathbb{R}^n \setminus \{x\})$ to derive the homology groups of the pair $H_i(\mathbb{R}^n, \mathbb{R}^n \setminus \{x\})$ for all $i \geq 0$.

(b) (3 points) Let $n, m \geq 1$. Deduce that $\mathbb{R}^n$ is not homeomorphic to $\mathbb{R}^m$ for $n \neq m$.

[5] Just say "True" or "False". No justification is required; no partial credit is available.

(a) Let X be a space with $A \subset X$. The map $H_0(A) \to H_0(X)$ is injective iff each path-component of X contains at most one path-component of A.

(b) Given a chain complex $\ldots \to C_{k+1} \xrightarrow{d_{k+1}} C_k \xrightarrow{d_k} C_{k-1} \to \ldots$, we have a short exact sequence $0 \to \ker d_n \to C_n \to \operatorname{im} d_n \to 0$ for all n.

(c) For $A, B \subset X$ with $X = \operatorname{int} A \cup \operatorname{int} B$, the inclusion $(B, A \cap B) \hookrightarrow (X, A)$ induces isomorphisms $H_i(B, A \cap B) \to H_i(X, A)$ for all i.

(d) If the spaces X and Y satisfy $H_i(X) \cong H_i(Y)$ for all $i \geq 0$, then X and Y are homotopy equivalent.

(e) If a map $f \colon S^n \to S^n$ is surjective, then $\deg(f) \neq 0$.

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