UF MTG 6347 **Exam 2**

Name: ____

- Explain your work (efficiently); partial credit is available.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:
  "I will not give, receive, or use any unauthorized assistance."

Signature: ____

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

1 Prove there is no map $\mathbb{R}P^n \to \mathbb{R}P^m$ inducing a nontrivial map $H^1(\mathbb{R}P^m; \mathbb{Z}_2) \to H^1(\mathbb{R}P^n; \mathbb{Z}_2)$ if $n > m$. What is the corresponding result for maps $\mathbb{C}P^n \to \mathbb{C}P^m$? [*Do not prove the $\mathbb{C}P^n$ version.*]

2 For M a closed n-dimensional manifold, prove $H_n(M, M - \{x\}; R) \cong R$.

3 Let M_1 and M_2 be two closed connected n-manifolds, and let $M_1 \# M_2$ be their connected sum (obtained by deleting the interiors of n-balls from each and identifying the resulting boundary spheres). Show there are isomorphisms $H_i(M_1 \# M_2; \mathbb{Z}) \cong H_i(M_1; \mathbb{Z}) \oplus H_i(M_2; \mathbb{Z})$ for all $0 < i < n-1$.

[*Note you are* not *asked to treat the exceptional case* $i = n - 1$.]

4 Let X be a space and let R be a commutative ring. We defined a cap product $C_k(X;R) \times C^l(X;R) \xrightarrow{\cap} C_{k-l}(X;R)$, and showed that for $\sigma \in C_k(X;R)$ and $\varphi \in C^l(X;R)$, we have

$$\partial(\sigma \cap \varphi) = (-1)^l(\partial\sigma \cap \varphi - \sigma \cap \delta\varphi).$$

Use the above formula to prove that we get an induced map

$$H_k(X;R) \times H^l(X;R) \xrightarrow{\cap} H_{k-l}(X;R).$$

5 Just say "True" or "False". No justification is required; no partial credit is available.

(a) There exists a closed orientable 6-dimensional manifold M with $H^3(M;\mathbb{Z}) \cong \mathbb{Z}$.

(b) If $0 \to A \to B \to C \to 0$ is a short exact sequence of finitely generated abelian groups, then $\operatorname{rank}(B) = \operatorname{rank}(A) + \operatorname{rank}(C)$.

(c) Let N_g be the nonorientable surface of genus g, which has $H^1(N_g;\mathbb{Z}_2) \cong (\mathbb{Z}_2)^g$. Then there is a generating set $\varphi_1, \ldots, \varphi_g$ for $H^1(N_g;\mathbb{Z}_2) \cong (\mathbb{Z}_2)^g$ with $\varphi_i \cup \varphi_j = 0$ for all $i \neq j$.

(d) Let M be a closed n-dimensional manifold and let R be a commutative ring with identity. Then M is R-orientable if and only if a fundamental class $[M] \in H_n(M;R)$ exists.

(e) Let M_g be the orientable surface of genus g. The Euler characteristic of $M_g \times \mathbb{R}P^3$ is zero.

This page intentionally left blank.

This page intentionally left blank.