UF MTG 4303/5317 **Final**

Name: ________________________________

- Explain your work (efficiently); partial credit is available.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:
  "I will not give, receive, or use any unauthorized assistance."

Signature: ____

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

$\boxed{1}$ Let f be a loop in X based at $x_0 \in X$. Show that $f * e_{x_0} \simeq_p f$ by *explicitly* writing down a path homotopy $H\colon I \times I \to X$ between $f * e_{x_0}$ and f.

$\boxed{2}$ Let α be a path in X from x_0 to x_1. Define the map $\hat{\alpha}\colon \pi_1(X, x_0) \to \pi_1(X, x_1)$.

Show that if $\pi_1(X, x_0)$ is abelian, then $\hat{\alpha} = \hat{\beta}$ for every pair of paths α and β from x_0 to x_1.

$\boxed{3}$ Let E and B be topological spaces. Define what it means for a function $p\colon E \to B$ to be a *covering map*.

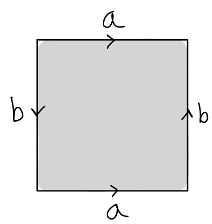
Prove that if $p\colon E \to B$ and $p'\colon E' \to B'$ are covering maps, then $p \times p'\colon E \times E' \to B \times B'$ defined by $(p \times p')(e, e') = (p(e), p'(e'))$ is a covering map.

$\boxed{4}$ Let X be a topological space, let $A \subset X$, let $i\colon A \to X$ be the inclusion map, and let $r\colon X \to A$ be continuous. Define what it means for r to be a *retraction*. Define what it means for r to be a *deformation retraction*. Give an example of $r\colon X \to A$ such that r is a retraction but not a deformation retraction (you do not need to prove your answer is correct).

$\boxed{5}$ Use the fact that there is no continuous antipode-preserving map $g\colon S^2 \to S^1$ in order to prove the Borsuk–Ulam theorem for S^2: "*If $f\colon S^2 \to \mathbb{R}^2$ is continuous, then there is a point $x \in S^2$ with $f(x) = f(-x)$.*"

$\boxed{6}$ Let $B^2 = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$ be the unit disk. Prove the Brouwer fixed point theorem: *"Any continuous map $f\colon B^2 \to B^2$ has a fixed point $x \in B^2$ with $f(x) = x$."*

$\boxed{7}$ Use the Seifert-van Kampen theorem to compute the fundamental group $\pi_1(K)$, where K is the Klein bottle, defined as the identification space drawn below.



$\boxed{8}$ Give the precise definition of a topological space X whose fundamental group is isomorphic to $\mathbb{Z}/2$. Then, describe a topological space X whose fundamental group is isomorphic to $\mathbb{Z}/2 \times \mathbb{Z}/2$. (You do not need to prove either answer is correct.)

$\boxed{9}$ The connected sum of two surfaces S and S' is obtained by cutting out a disk from each surface and gluing the resulting surfaces along their circle boundaries: $S\#S' = (S\setminus D^2)\cup_{S^1}(S'\setminus D^2)$. Let T be the torus. Using the Seifert-van Kampen theorem *on the natural decomposition into two pieces given by the connected sum*, calculate $\pi_1(T\#T)$.

$\boxed{10}$ Let X be the quotient space obtained from an 8-sided polygonal region P by pasting its edges together according to the labelling scheme $abcdb^{-1}c^{-1}a^{-1}d^{-1}$. It turns out that all vertices of P are mapped to the same point of the quotient space X by the pasting map. Calculate $H_1(X)$, and using this, determine which compact surface X is homeomorphic to.

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