

UF MTG 6346  
Topology 1  
Fall 2024

## Homework 1

Due Friday, August 30, anytime, on Canvas

**Reading.** Hatcher pages 1–17.

**Problems.**

1. Chapter 0, Exercise 1. Construct an explicit deformation retraction of the torus with one point deleted onto a graph consisting of two circles intersecting in a point, namely, longitude and meridian circles of the torus.

*Remark: “Explicit” means give formulas.*

*Hint: I recommend you parameterize your torus as a square in the plane with opposite sides identified.*

2. Chapter 0, Exercise 3.

(a) Show that the composition of homotopy equivalences  $X \rightarrow Y$  and  $Y \rightarrow Z$  is a homotopy equivalence  $X \rightarrow Z$ . Deduce that homotopy equivalence is an equivalence relation.

(b) Show that the relation of homotopy among maps  $X \rightarrow Y$  is an equivalence relation.

(c) Show that a map homotopic to a homotopy equivalence is a homotopy equivalence.

3. Chapter 0, Exercise 16. Show that  $S^\infty$  is contractible.

4. Chapter 0, Exercise 19. Show that the space obtained from  $S^2$  by attaching  $n$  2-cells along any collection of  $n$  circles in  $S^2$  is homotopy equivalent to the wedge sum of  $n+1$  2-spheres.

**Recommend Problems (not to turn in).**

- Chapter 0, Exercise 2.
- Chapter 0, Exercise 6.
- Chapter 0, Exercise 9.