UF MTG 6347
Topology 2
Spring 2025

Homework 1

Due Friday, January 31, anytime, on Canvas

Reading. Hatcher pages 160–165, 177–182, 185–205.

Problems.

1. §2.3, Exercise 1. If $T_n(X, A)$ denotes the torsion subgroup of $H_n(X, A; \mathbb{Z})$, show that the functors $(X, A) \mapsto T_n(X, A)$, with the obvious induced homomorphisms $T_n(X, A) \to T_n(Y, B)$ and boundary maps $T_n(X, A) \to T_{n-1}(A)$, do not define a homology theory. Do the same for the 'mod torsion' functor $MT_n(X, A) = H_n(X, A; \mathbb{Z})/T_n(X, A)$.

2. §2.C, Exercise 1. What is the minimum number of edges in simplicial complex structures K and L on S^1 such that there is a simplicial map $K \to L$ of degree n?

3. §2.C, Exercise 2. Use the Lefschetz fixed point theorem to show that a map $S^n \to S^n$ has a fixed point unless its degree is equal to the degree of the antipodal map $x \mapsto -x$.

4. §2.C, Exercise 5. Let M be a closed orientable surface embedded in $\mathbb{R}^3$ in such a way that reflection across a plane P defines a homeomorphism $r\colon M \to M$ fixing $M \cap P$, a collection of circles. Is it possible to homotope r to have no fixed points?