UF MTG 4302/5316
Introduction to Topology 1
Fall 2023

Homework 2

Due Monday, September 18, anytime, on Canvas

Reading. Munkres §10 – §15

Problems.

- §9 #5.

  (a) Use the choice axiom to show that if $f\colon A \to B$ is surjective, then f has a right inverse $h\colon B \to A$.

  (b) Show that if $f\colon A \to B$ is injective and A is not empty, then f has a left inverse. Is the axiom of choice needed?

  If $f\colon A \to B$ is a function, then a right inverse *is a function $h\colon B \to A$ satisfying $f(h(b)) = b$ for all $b \in B$, and a* left inverse *is a function $g\colon B \to A$ satisfying $g(f(a)) = a$ for all $a \in A$.*

- §11 #8. A typical use of Zorn's lemma in algebra is the proof that every vector space has a basis. Recall that if A is a subset of the vector space V, we say a vector belongs to the *span* of A if it equals a finite linear combination of elements of A. The set A is *independent* if the only finite linear combination of elements of A that equals the zero vector is the trivial one having all coefficients zero. If A is independent and if every vector in V belongs to the span of A, then A is a *basis* for V.

  (a) If A is independent and $v \in V$ does not belong to the span of A, show $A \cup \{v\}$ is independent.

  (b) Show the collection of all independent sets in V has a maximal element.

  (c) Show that V has a basis.

- §13 #4(a). If $\{\tau_\alpha\}$ is a family of topologies on X, show that $\cap\tau_\alpha$ is a topology on X. Is $\cup\tau_\alpha$ a topology on X?

Recommend Problems (not to turn in).

- §9 #6.
- §11 #2; this question shows that partially ordered sets defined using the axioms for $\prec$ are really the same mathematical object as partially ordered sets defined using the axioms for $\preceq$.
- §13 #1.