UF MTG 4303/5317
Introduction to Topology 2
Spring 2024

Homework 2

Due Monday, February 5, anytime, on Canvas

Reading. Munkres §54–55, 35, 38.

Problems.

1. §51 #3(a,b,c). A space X is said to be *contractible* if the identity map $i_X\colon X \to X$ is nullhomotopic.
   (a) Show that I and $\mathbb{R}$ are contractible.
   (b) Show that a contractible space is path connected.
   (c) Show that if Y is contractible, then for any X, the set $[X, Y]$ has a single element.

   Remark: Note that $[X, Y]$ is defined in the prior problem, §51 #2.

2. §52 #2. Let α be a path in X from x_0 to x_1; let β be a path in X from x_1 to x_2. Show that if $\gamma = \alpha * \beta$, then $\hat{\gamma} = \hat{\beta} \circ \hat{\alpha}$.

3. §53 #3. Let $p\colon E \to B$ be a covering map; let B be connected. Show that if $p^{-1}(b_0)$ has k elements for some $b_0 \in B$, then $p^{-1}(b)$ has k elements for every $b \in B$. In such a case, E is called a *k-fold covering* of B.

4. §54 #3. Let $p\colon E \to B$ be a covering map. Let α and β be paths in B with $\alpha(1) = \beta(0)$; let $\tilde{\alpha}$ and $\tilde{\beta}$ be liftings of them such that $\tilde{\alpha}(1) = \tilde{B}(0)$. Show that $\tilde{\alpha} * \tilde{\beta}$ is a lifting of $\alpha * \beta$.

Recommend Problems (not to turn in).

- §52 #1
- §54 #1