UF MTG 6346
Topology 1
Fall 2024

Homework 2

Due Friday, September 20, anytime, on Canvas

Reading. Hatcher §1.1–1.2, pages 21–55.

Problems.

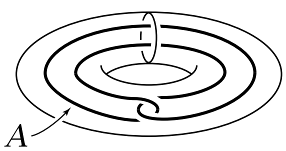
1. §1.1, Exercise 3. For a path-connected space X, show that $\pi_1(X)$ is abelian iff all basepoint-change homomorphisms β_h depend only on the endpoints of the path h.

2. §1.1, Exercise 16(a,b,c,f). Show that there are no retractions $r\colon X \to A$ in the following cases:

   (a) $X = \mathbb{R}^3$ with A any subspace homeomorphic to S^1.

   (b) $X = S^1 \times D^2$ with A its boundary torus $S^1 \times S^1$.

   (c) $X = S^1 \times D^2$ with A the circle shown in the figure.

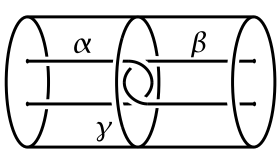
   (f) X the Möbius band and A its boundary circle.

3. §1.2, Exercise 8.

   (a) "*Compute the fundamental group of the space obtained from two tori* $S^1 \times S^1$ *by identifying a circle* $S^1 \times \{x_0\}$ *in one torus with the corresponding circle* $S^1 \times \{x_0\}$ *in the other torus.*" Let's call this identification space X. Compute $\pi_1(X)$ by using van Kampen's theorem on the natural union of two tori along a circle.

   (b) Write this identification space X as a product $X = Y \times Z$ (where neither Y nor Z are just a single point), and use this to give an alternate computation of $\pi_1(X)$.

4. §1.2, Exercise 10. Consider two arcs α and β embedded in $D^2 \times I$ as shown in the figure. The loop γ is obviously nullhomotopic in $D^2 \times I$, but show that there is no nullhomotopy of γ in the complement of $\alpha \cup \beta$.

   [Hint: What is the fundamental group of $(D^2 \times I) \setminus (\alpha \cup \beta)$*?]*

Recommend Problems (not to turn in).

- §1.1, Exercise 5.
- §1.1, Exercise 6.
- §1.1, Exercise 9.
- §1.2, Exercise 6.
- §1.2, Exercise 9.