UF MTG 6347
Topology 2
Spring 2025

Homework 2

Due Monday, February 10, anytime, on Canvas

Reading. Hatcher pages 190–205.

Problems.

1. Prove the Splitting Lemma on Page 147 of Hatcher.
2. §3.1, Exercise 5. Regarding a cochain $\phi \in C^1(X; G)$ as a function from paths in X to G, show that if ϕ is a cocycle, then

   (a) $\phi(f \cdot g) = \phi(f) + \phi(g)$,

   (b) ϕ takes the value 0 on constant paths,

   (c) $\phi(f) = \phi(g)$ if $f \simeq g$,

   (d) ϕ is a coboundary iff $\phi(f)$ depends only on the endpoints of f, for all f.
3. §3.1, Exercise 6(b). Directly from the definitions, compute the simplicial cohomology groups of $\mathbb{R}\mathrm{P}^2$ and the Klein bottle, each with $\mathbb{Z}$ and $\mathbb{Z}_2$ coefficients, using the Δ-complex structure given in §2.1.
4. §3.1, Exercise 8(a). Many basic homology arguments work just as well for cohomology even though maps go in the opposite direction. Verify this in the following case.

   (a) Compute $H^i(S^n; G)$ by induction on n in two ways: using the long exact sequence of a pair, and using the Mayer–Vietoris sequence.