UF MTG 4303/5317
Introduction to Topology 2
Spring 2024

Homework 3

Due Wednesday, February 28, anytime, on Canvas

Reading. Munkres §37, 56–59.

Problems.

1. §52 #3. Let x_0 and x_1 be points of the path-connected space X. Show that $\pi_1(X, x_0)$ is abelian if and only if for every pair α and β of paths from x_0 to x_1, we have $\hat{\alpha} = \hat{\beta}$.

2. §55 #4(d,e). Suppose that you are given the fact that for each n, there is no retraction $r\colon B^{n+1} \to S^n$. (This result can be proved using more advanced techniques of algebraic topology.) Prove the following:

   (d) Every continuous map $f\colon B^{n+1} \to B^{n+1}$ has a fixed point.

   (e) Every $n+1$ by $n+1$ matrix with positive real entries has a positive eigenvalue.

3. §57 #4(a,b,c). Suppose you are given the fact that for each n, no continuous antipode-preserving map $h\colon S^n \to S^n$ is nullhomotopic. (This result can be proved using more advanced techniques of algebraic topology.) Prove the following:

   (a) There is no retraction $r\colon B^{n+1} \to S^n$.

   (b) There is no continuous antipode-preserving map $g\colon S^{n+1} \to S^n$.

   (c) (Borsuk–Ulam theorem) Given a continuous map $f\colon S^{n+1} \to \mathbb{R}^{n+1}$, there is a point x of S^{n+1} such that $f(x) = f(-x)$.

Recommend Problems (not to turn in).

- §55 #4(a)
- §57 #1
- §57 #2