UF MTG 6346
Topology 1
Fall 2024

Homework 3

Due Monday, October 7, anytime, on Canvas

Reading. Hatcher pages 56–87.

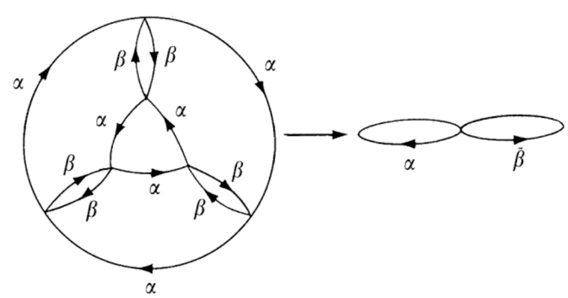
Problems.

1. Let M be an n-dimensional manifold, with $n \geq 3$. Let $p \in M$ be any point in the manifold M. There is a nice relationship between the fundamental groups $\pi_1(M)$ and $\pi_1(M \setminus \{p\})$ — how are they related? Prove your answer is correct.

2. §1.3, Exercise 9. Show that if a path-connected, locally path-connected space X has $\pi_1(X)$ finite, then every map $X \to S^1$ is nullhomotopic. [Use the covering space $\mathbb{R} \to S^1$.]

3. §1.3, Exercise 12. Let a and b be the generators of $\pi_1(S^1 \vee S^1)$ corresponding to the two S^1 summands. Draw a picture of the covering space of $S^1 \vee S^1$ corresponding to the normal subgroup generated by a^2, b^2, and $(ab)^4$, and prove that this covering space is indeed the correct one.

4. Let $\tilde{X}$ be the 6-fold cover of $S^1 \vee S^1$ drawn below.



   (a) Use Proposition 1.32 and Proposition 1.39 to deduce the size of the group $G(\tilde{X})$ of deck transformations. Use the symmetries of $\tilde{X}$ to identify the group $G(\tilde{X})$.

   (b) Alter $\tilde{X}$ by reversing the direction of the three α arrows on the inner circle only. What is the size of the group $G(\tilde{X})$ of deck transformations? What is the group $G(\tilde{X})$?

Recommend Problems (not to turn in).

- §1.3, Exercise 10.
- §1.3, Exercise 17.