UF MTG 4302/5316
Introduction to Topology 1
Fall 2023

Homework 4

Due Friday, October 20, anytime, on Canvas

Reading. Munkres §21 – §25

Problems.

- §18 #9(a,b). Let $\{A_\alpha\}$ be a collection of subsets of X; let $X = \cup_\alpha A_\alpha$. Let $f\colon X \to Y$; suppose that $f|A_\alpha$ is continuous for each α.

  (a) Show that if the collection $\{A_\alpha\}$ is finite and each set A_α is closed, then f is continuous.

  (b) Find an example where the collection $\{A_\alpha\}$ is countable and each A_α is closed, but f is not continuous.

- §19 #6. Let $\mathbf{x}_1, \mathbf{x}_2, \ldots$ be a sequence of points of the product space $\prod X_\alpha$. Show that this sequence converges to the point $\mathbf{x}$ if and only if the sequence $\pi_\alpha(\mathbf{x}_1), \pi_\alpha(\mathbf{x}_2), \ldots$ converges to $\pi_\alpha(\mathbf{x})$ for each α. Is this fact true if one uses the box topology instead of the product topology?

- §21 #3. Let X_n be a metric space with metric d_n, for $n \in \mathbb{Z}_+$.

  (a) Show that $\rho(x, y) = \max\{d_1(x_1, y_1), \ldots, d_n(x_n, y_n)\}$ is a metric for the product space $X_1 \times \ldots \times X_n$.

  (b) Let $\overline{d_i} = \min\{d_i, 1\}$. Show that $D(x, y) = \sup\left\{\frac{\overline{d_i}(x_i, y_i)}{i}\right\}$ is a metric for the product space $\prod X_i$.

  Remark: For (b), compare the proof of Theorem 20.5, which is the specific case when $X_n = \mathbb{R}$ and d_n is the standard metric on $\mathbb{R}$, for all n.

- §22 #2(a). Let $p\colon X \to Y$ be a continuous map. Show that if there is a continuous map $f\colon Y \to X$ such that $p \circ f$ equals the identity map of Y, then p is a quotient map.

Recommend Problems (not to turn in).

- §17 #13.
- §19 #1.
- §20 #4.