UF MTG 6346
Topology 1
Fall 2024

Homework 4

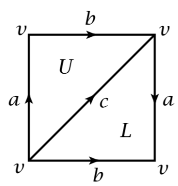
Due Wednesday, October 30, anytime, on Canvas

Reading. Hatcher pages 97–133.

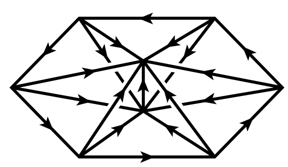
Problems.

1. §2.1, Exercise 4. Compute the simplicial homology groups of the triangular parachute obtained from Δ^2 by identifying its three vertices to a single point.

2. §2.1, Exercise 5. Compute the simplicial homology groups of the Klein bottle using the Δ-complex structure described at the beginning of this section.



3. §2.1, Exercise 8. Construct a 3-dimensional Δ-complex X from n tetrahedra $T_1, \ldots, T_n$ by the following steps. First arrange the tetrahedra in a cyclic pattern as in the figure, so that each T_i shares a common vertical face with its two neighbors T_{i-1} and T_{i+1}, subscripts being taken mod n. Then identify the bottom face of T_i with the top face of T_{i+1} for each i. Show the simplicial homology groups of X in dimensions $0, 1, 2, 3$ are $\mathbb{Z}, \mathbb{Z}_n, 0, \mathbb{Z}$, respectively. [The space X is an example of a *lens space*; see Example 2.43 for the general case.]



4. §2.1, Exercise 11. Show that if A is a retract of X then the map $H_n(A) \to H_n(X)$ induced by the inclusion $A \subset X$ is injective.

Recommend Problems (not to turn in).

- Compute the simplicial homology of the Klein bottle with $\mathbb{Z}/2\mathbb{Z}$ coefficients (defined on pages 153–154 of Hatcher).

- §2.1, Exercise 3.