UF MTG 4302/5316
Introduction to Topology 1
Fall 2023

Homework 5

Due Wednesday, November 1, anytime, on Canvas

Reading. Munkres §26 – §32

Problems.

- §23 #9. Let A be a proper subset of X, and let B be a proper subset of Y. If X and Y are connected, show that $(X \times Y) - (A \times B)$ is connected.

  Remark: This problem has appeared on the first year exam before; see for example ([1]).

- §24 #1(a,c).

  (a) Show that no two of the spaces $(0, 1)$, $(0, 1]$, and $[0, 1]$ are homeomorphic.
  Hint: What happens if you remove a point from each of these spaces?

  (c) Show that $\mathbb{R}^n$ and $\mathbb{R}$ are not homeomorphic if $n > 1$.

- §26 #3. Show that a finite union of compact subspaces of X is compact.

- §28 #7(a,b). Let (X, d) be a metric space. If f satisfies the condition $d(f(x), f(y)) < d(x, y)$ for all $x, y \in X$ with $x \neq y$, then f is called a *shrinking map*. If there is a number $\alpha < 1$ such that $d(f(x), f(y)) \leq \alpha d(x, y)$ for all $x, y \in X$, then f is called a *contraction*. A *fixed point* of f is a point x such that $f(x) = x$.

  (a) If f is a contraction and X is compact, show that f has a unique fixed point.

  (b) Show more generally that if f is a shrinking map and X is compact, then f has a unique fixed point.

  Remark: This "Contraction mapping theorem" problem is on the first year exam syllabus: ([2]). See our textbook for some hints, though those hints don't need to be followed.

Recommend Problems (not to turn in).

- §25 #3.
- §27 #4.
- §28 #6.

[1] `https://gma.math.ufl.edu/gma/wp-content/uploads/sites/130/FY_Topology_2013_01_part1.pdf`

[2] `https://math.ufl.edu/first-year-exam-syllabi/mtg-5316-introduction-to-topology-1/`