UF MTG 4303/5317
Introduction to Topology 2
Spring 2024

Homework 5

Due Friday, April 5, anytime, on Canvas

Reading. Munkres §69–72.

Problems.

1. §68 #2. Let $G = G_1 * G_2$, where G_1 and G_2 are nontrivial groups.

   (a) Show G is not abelian.

   (b) If $x \in G$, define the *length* of x to be the length of the unique reduced word in the elements of G_1 and G_2 that represents x. Show that if x has even length (at least 2), then x does not have finite order. Show that if x has odd length (at least 3), then x is conjugate to an element of shorter length.

   (c) Show that the only elements of G that have finite order are the elements of G_1 and G_2 that have finite order, and their conjugates.

2. Let (Y, y_0) and (Z, z_0) be path-connected pointed spaces such that Y has a contractible neighborhood $N_Y \ni y_0$ and Z has a contractible neighborhood $N_Z \ni z_0$. Prove that $\pi_1(Y \vee Z)$ is isomorphic to $\pi_1(Y) * \pi_1(Z)$.

   [Recall that the *wedge sum* of two pointed spaces (Y, y_0) and (Z, z_0) is the quotient space $(Y \coprod Z)/(y_0 \sim z_0)$.]

3. The connected sum of two surfaces S and S' is obtained by cutting out a disk from each surface and gluing the resulting surfaces along their circle boundaries: $S\#S' = (S \setminus D^2) \cup_{S^1} (S' \setminus D^2)$. Let T be the torus. Using the Seifert-van Kampen theorem on the natural decomposition into two pieces given by the connected sum, calculate $\pi_1(T\#T)$.

Recommend Problems (not to turn in).

- §59 #2
- §60 #2
- §68 #3
- §70 #3