UF MTG 6346
Topology 1
Fall 2024

Homework 5

Due Friday, November 8, anytime, on Canvas

Reading. Hatcher pages 113–137.

Problems.

1. (a) Pick a Δ-complex structure on the n-dimensional ball D^n for $n \geq 1$. This induces a Δ-complex structure on its boundary $(n-1)$-sphere S^{n-1}. Compute the simplicial relative homology $H_n^{\Delta}(D^n, S^{n-1})$ from its definition (i.e., the homology of the chain complex $\ldots \to \Delta_{i+1}(D^n, S^{n-1}) \to \Delta_i(D^n, S^{n-1}) \to \Delta_{i-1}(D^n, S^{n-1}) \to \ldots$).

   (b) Compute the singular relative homology $H_n(D^n, S^{n-1})$ for $n \geq 1$ by using the LES for a pair of spaces (on pages 115 and 117 of Hatcher).

2. The proof of Theorem 2.16 in Hatcher contains 6 verifications of inclusions. Prove them. For one of the last three such inclusions ($\ker j_* \subseteq \mathrm{im} i_*$ or $\ker \partial \subseteq \mathrm{im} j_*$ or $\ker i_* \subseteq \mathrm{im}\partial$, your choice of which one), draw a diagram of the SES of chain complexes $0 \to A_\bullet \to B_\bullet \to C_\bullet \to 0$, and show where all the elements you consider live in this diagram.

3. §2.1, Exercise 16.

   (a) Show that $H_0(X, A) = 0$ iff A meets each path-component of X.

   (b) Show that $H_1(X, A) = 0$ iff $H_1(A) \to H_1(X)$ is surjective and each path-component of X contains at most one path-component of A.

   Remark: For (b), Exercise 15 of Hatcher is useful. It says that if $A \to B \to C \to D \to E$ is an exact sequence, then $C = 0$ iff the map $A \to B$ is surjective and $D \to E$ is injective.

4. §2.1, Exercise 20. Show that $\tilde{H}_n(X) \cong \tilde{H}_{n+1}(SX)$ for all n, where SX is the suspension of X. More generally, thinking of SX as the union of two cones CX with their bases identified, compute the reduced homology groups of the union of any finite number of cones CX with their bases identified.

Recommend Problems (not to turn in).

- §2.1, Exercise 17.
- §2.1, Exercise 22.