UF MTG 4302/5316
Introduction to Topology 1
Fall 2023

Homework 6

Due Monday, November 20, anytime, on Canvas

Reading. Munkres §32 – §33, §43.

Problems.

1. Define what it means for a topological space to be regular.

   Show that every compact Hausdorff space is regular.

   Remark: It is furthermore true that every compact Hausdorff space is normal (Theorem 32.3), which suffices since every normal space is regular. Read Theorem 32.3, but don't use it. Also don't use Lemma 26.4 — instead, I'm asking you to use the proof technique of Lemma 26.4 to reprove a variant thereof.

   Remark: This problem has appeared on the first year exam before; see for example ([1]).

2. Assume that $f\colon X \to Y$ is continuous and surjective.

   (a) If X is Lindelöf, show that Y is also.

   (b) If X is separable, show that Y is also.

   Remark: Recall from page 192 of Munkres that a space is separable *if it has a countable dense subset. So for a space, "being separable" (page 192) means something very different than "having a separation" (page 148).*

   Remark: This problem has appeared on the first year exam before; see for example ([2]).

Recommend Problems (not to turn in).

- §30 #4.
- §32 #1.

[1] https://gma.math.ufl.edu/wp-content/uploads/sites/130/FY-Topology-1-2023_08.pdf
[2] https://gma.math.ufl.edu/wp-content/uploads/sites/130/FY-Topology-1-2022_08.pdf