UF MTG 6346
Topology 1
Fall 2024

Homework 6

Due Friday, November 22, anytime, on Canvas

Reading. Hatcher pages 134–159.

Problems.

1. §2.2, Exercise 1. Prove the Brouwer fixed point theorem for maps $f\colon D^n \to D^n$ by applying degree theory to the map $S^n \to S^n$ that sends both the northern and southern hemispheres of S^n to the southern hemisphere via f. [This was Brouwer's original proof.]

2. §2.2, Exercise 9. Compute the homology groups of the following 2-complexes:

   (a) The quotient of S^2 obtained by identifying the north and south poles to a point. *Remark:* For pedagogical purposes, I recommend using the long exact sequence for the singular homology of the pair (S^2, S^0).

   (c) The space obtained from D^2 by first deleting the interiors of two disjoint subdisks in the interior of D^2 and then identifying all three resulting boundary circles together via homeomorphisms preserving clockwise orientations of these circles.

3. §2.2, Exercise 32. For SX the suspension of X, show by a Mayer–Vietoris sequence that there are isomorphisms $\tilde{H}_n(SX) \cong \tilde{H}_{n-1}(X)$ for all n.

4. Consider the CW-complex structure on real projective space $\mathbb{R}\mathrm{P}^n$ that has a single k-cell for each $0 \leq k \leq n$.

   (a) Using this CW structure, compute the cellular homology of $\mathbb{R}\mathrm{P}^n$. *Remark: Some steps look different for the case of n odd versus n even.*

   (b) Compute the homology of $\mathbb{R}\mathrm{P}^n$ with $\mathbb{Z}_2$ coefficients (compute $H_i(\mathbb{R}\mathrm{P}^n; \mathbb{Z}_2)$).

   (c) Compute the homology of $\mathbb{R}\mathrm{P}^n$ with $\mathbb{Z}_3$ coefficients (compute $H_i(\mathbb{R}\mathrm{P}^n; \mathbb{Z}_3)$).

Recommend Problems (not to turn in).

- Prove that S^n has a continuous nonzero vector field iff n is odd.
- Prove that $\mathbb{Z}_2$ is the only nontrivial group that can act freely on S^n if n is even.
- §2.2, Exercise 12.
- §2.2, Exercise 29.