UF MTG 4302/5316
Introduction to Topology 1
Fall 2023

Homework 7

Due Monday, December 4, anytime, on Canvas

Reading. Munkres §43–44, §48, §50.

Problems.

1. Prove that every compact Hausdorff space is normal. You may follow the proof of Theorem 32.3 in the book (which states this result).
2. §43 #4(forward direction). Show that if the metric space (X, d) is complete, then for every nested sequence $A_1 \supset A_2 \supset \dots$ of nonempty closed sets of X such that $\text{diam} A_n \to 0$, the intersection of the sets A_n is nonempty.
3. §44 #1. Given n, show there is a continuous surjective map $g\colon I \to I^n$. (*Hint:* Consider $f \times f\colon I \times I \to I^2 \times I^2$, where $f\colon I \to I^2$ is the surjective map from Theorem 44.1.)
4. §48 #4. Show that if every point x of X has a neighborhood that is a Baire space, then X is a Baire space. (*Hint:* Use the open set formulation of the Baire condition.)

   Remark: This problem has appeared on the first year exam before; see for example ([1]).

Recommend Problems (not to turn in).

- §32 #2.
- §43 #5.
- §44 #2.
- §48 #11.

[1] https://gma.math.ufl.edu/wp-content/uploads/sites/130/FY-Topology-1-2022_05.pdf