UF MTG 4303/5317 **Midterm**

Name: ________________________________

- Explain your work (efficiently); partial credit is available.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:
  "I will not give, receive, or use any unauthorized assistance."

Signature: ____

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

1 Let X be a topological space, and let $A \subset X$. Define what it means for a continuous map $r\colon X \to A$ to be a *retraction*.

Prove that if $r\colon X \to A$ is a *retraction*, then the induced map $r_*\colon \pi_1(X, x_0) \to \pi_1(A, x_0)$ is surjective for all $x_0 \in A$.

2 Use the fact that there is no retraction $r\colon B^2 \to S^1$ in order to prove the Brouwer fixed point theorem for the disk $B^2 = \{x \in \mathbb{R}^2 \ : \ \|x\| \leq 1\}$: "*If* $f\colon B^2 \to B^2$ *is continuous, then there is a point* $x \in B^2$ *with* $f(x) = x$."

3 Define what it means for a topological space X to be *simply connected.*

Let X be a simply connected space, and let $x_0, x_1 \in X$. Prove that if f and g are two paths in X from x_0 to x_1, then f and g are path homotopic.

4 Let E and B be topological spaces. Define what it means for a function $p\colon E \to B$ to be a *covering map*.

Let $p\colon E \to B$ be a covering map; let B be connected. Show that if $p^{-1}(b_0)$ has k elements for some $b_0 \in B$, then $p^{-1}(b)$ has k elements for every $b \in B$.

5 Let x_0 and x_1 be points of the path-connected space X. Show that if $\pi_1(X, x_0)$ is abelian, then $\hat{\alpha} = \hat{\beta}$ for every pair of paths α and β from x_0 to x_1.

(Recall $\hat{\alpha}$ and $\hat{\beta}$ are maps $\pi_1(X, x_0) \to \pi_1(X, x_1)$.)

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