UF MTG 4302/5316 **Practice Exam 1**

Name: ________________________________

- Explain your work (efficiently); partial credit is available.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:
  "I will not give, receive, or use any unauthorized assistance."

Signature: ____

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

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1. Let X be a topological space, and let $\mathcal{A}$ be a collection of open sets in X such that for each open set U in X and for each $x \in U$, there is an element A of $\mathcal{A}$ such that $x \in A \subset U$. Prove that $\mathcal{A}$ is a basis for the topology on X.

   Remark: I am not asking you to say "This is a theorem in our book; I am asking you to prove this theorem."

2 Define what it means for a topological space X to be *Hausdorff*. If X is Hausdorff, then prove that a sequence in X can converge to at most one point in X.

3 Let X and Y be topological spaces, and let $f\colon X \to Y$ be a function. Suppose that for each closed set C in Y, the preimage $f^{-1}(C)$ is closed in X. Prove that f is continuous.

4 Let X be a set and let $\mathcal{P}(X)$ be its power set (the set of all subsets of X). Prove that there is no surjective function $f\colon X \to \mathcal{P}(X)$.

5 Prove that each of the following sets is countable.

(a) $\mathbb{Z}_+ \times \mathbb{Z}_+$

(b) $\mathbb{Q}_+ = \{\frac{a}{b} \mid a, b \in \mathbb{Z}_+\}$

(c) The set J of all two-element subsets of $\mathbb{Z}_+$.

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