UF MTG 6346 **Practice Exam 1**

Name: ____

- Explain your work (efficiently); partial credit is available.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:
  "I will not give, receive, or use any unauthorized assistance."

Signature: ____

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

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1. Show that there is no retraction from the disk $D^2 = \{x \in \mathbb{R}^2 \ : \ \|x\| \leq 1\}$ to its boundary circle $S^1 = \{x \in \mathbb{R}^2 \ : \ \|x\| = 1\}$.

2 Let X be the union of path-connected open sets A_α each containing the basepoint $x_0 \in X$. Suppose each two-fold intersection $A_\alpha \cap A_\beta$ is path-connected, and furthermore each three-fold intersection $A_\alpha \cap A_\beta \cap A_\gamma$ is path-connected. Then van Kampen's Theorem provides a group that $\pi_1(X)$ is isomorphic to. What is this group?

3 Let $p\colon S^1 \times \mathbb{R} \to S^1 \times S^1$ be the covering space map defined by $p(e^{2\pi i \cdot s}, t) = (e^{2\pi i \cdot 3s}, e^{2\pi i \cdot t})$ for $e^{2\pi i \cdot s} \in S^1$ and $t \in \mathbb{R}$.

(a) Is this covering space normal? Identify the group of deck transformations.

(b) Verify that the group of deck transformations is isomorphic to the quotient $N(H)/H$, where $H = p_*(\pi_1(S^1 \times \mathbb{R}))$.

4 Let (X, x_0), $(\tilde{X}_1, \tilde{x}_1)$, and $(\tilde{X}_2, \tilde{x}_2)$ be path-connected and locally path-connected spaces with distinguished basepoints. Let $p_1 \colon (\tilde{X}_1, \tilde{x}_1) \to (X, x_0)$ and $p_2 \colon (\tilde{X}_2, \tilde{x}_2) \to (X, x_0)$ be basepoint-preserving covering space maps.

- (3 points) Define what it means for $f \colon (\tilde{X}_1, \tilde{x}_1) \to (\tilde{X}_2, \tilde{x}_2)$ to be an *isomorphism of covering spaces*.
- (7 points) Prove that the covering spaces p_1 and p_2 are isomorphic if and only if $p_{1*}(\pi_1(\tilde{X}_1, \tilde{x}_1)) = p_{2*}(\pi_1(\tilde{X}_2, \tilde{x}_2))$.

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5 Just say "True" or "False". No justification is required; no partial credit is available.

(a) There exists a CW complex structure on $\mathbb{R}P^n$ with one i-cell for each $0 \leq i \leq n$.

(b) The Möbius band is a mapping cylinder.

(c) Let G be an arbitrary group. Then G can be obtained as the fundamental group of some CW complex.

(d) There exists a 4-dimensional CW complex X with no 2-cells such that $\pi_1(X) \cong \mathbb{Z} * (\mathbb{Z}/3\mathbb{Z})$.

(e) Let $\tilde{X}$ be a torus. Then there exists a space X and a covering space map $p\colon \tilde{X} \to X$ with $p_*(\pi_1(\tilde{X})) \cong \mathbb{Z}$.

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