**UF MTG 6347** **Practice Exam 1**

Name: ______________________________

- Explain your work (efficiently); partial credit is available.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:
  "I will not give, receive, or use any unauthorized assistance."

Signature: ______________________________________

| Problem | Total Points | Score |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| Total | 50 | |

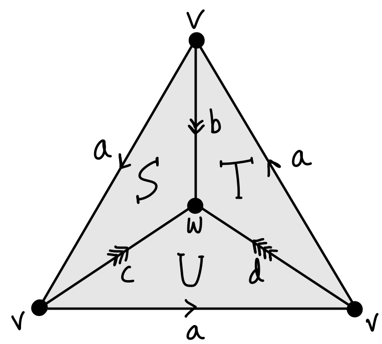
**1** Let $X$ and $Y$ be spaces and let $G$ be an abelian group. A *cochain map* is a collection of maps $f^{\#}\colon C^n(Y;G) \to C^n(X;G)$ for all $n$, such that $f^{\#}\delta_Y^n = \delta_X^n f^{\#}$ for all $n$.

*Remark: Here $C^n(Y;G)$ is the singular cochain group in $Y$, and $\delta_Y^n$ is a coboundary map for $Y$, and $\delta_X^n$ is a coboundary map for $X$.*

(a) Show that a cochain map $f^{\#}\colon C^n(Y;G) \to C^n(X;G)$ for all $n$ induces a map $f^*\colon H^n(Y;G) \to H^n(X;G)$ on singular cohomology groups for all $n$.

(b) Given two cochain maps $f^{\#}, g^{\#}\colon C^n(Y;G) \to C^n(X;G)$, a *cochain homotopy* between them is a collection of maps $P^n\colon C^n(Y;G) \to C^{n-1}(X;G)$ for all $n$ that satisfy $g^{\#} - f^{\#} = P^{n+1}\delta_Y^n + \delta_X^{n-1}P^n$. Prove that if there is a chain homotopy between $f^{\#}$ and $g^{\#}$, then $f^{\#}$ and $g^{\#}$ induce the same maps $f^* = g^*$ on cohomology.

2 Let $X$ be the $\Delta$-complex drawn below, which has two 0-simplices $v$ and $w$, four 1-simplices $a, b, c, d$, and three 2-simplices $S, T, U$. Compute the simplicial cohomology $H^i(X;\mathbb{Z})$ for $i = 0, 1, 2$.



3 Let $X$ be a space and let $R$ be a ring. Let $C^k(X;R)$ be the group of singular $k$-cochains on $X$. We defined a cup product $\cup\colon C^k(X;R) \times C^l(X;R) \to C^{k+l}(X;R)$, and showed that for $f \in C^k(X;R)$ and $g \in C^l(X;R)$, we have

$$\delta(f \cup g) = (\delta f \cup g) + (-1)^k (f \cup \delta g).$$

Use the above formula to prove that we get an induced map on cohomology

$$\cup\colon H^k(X;R) \times H^l(X;R) \to H^{k+l}(X;R).$$

4 Using the cup product structure, show there is no map $\mathbb{R}P^n \to \mathbb{R}P^m$ inducing a nontrivial map $H^1(\mathbb{R}P^n;\mathbb{Z}_2) \to H^1(\mathbb{R}P^n;\mathbb{Z}_2)$ if $n > m$. What is the corresponding result for maps $\mathbb{C}P^n \to \mathbb{C}P^m$?

*Remark: This is Hatcher §3.2, Exercise 3(a).*

5 Just say "True" or "False". No justification is required; no partial credit is available.

(a) If $K$ and $L$ are simplicial complexes with $K$ finite, then any map $f\colon |K| \to |L|$ is homotopic to a simplicial map $K \to L$.

(b) If the spaces $X$ and $Y$ satisfy $H_i(X;\mathbb{Z}) \cong H_i(Y;\mathbb{Z})$ for all $i \geq 0$, then $X$ and $Y$ are homotopy equivalent.

(c) Let $X$ be a finite simplicial complex and $f\colon X \to X$. If

$$\tau(f) \coloneqq \sum_n (-1)^n \operatorname{tr}(f_*\colon H_n(X) \to H_n(X))$$

is zero, then $f$ has no fixed points.

(d) For $X$ a $\Delta$-complex with $k$-simplex $\sigma$, the dual cochain $\sigma^*$ is a cocycle if $\sigma$ is maximal.

(e) Let $C$ be a chain complex of free abelian groups. If $\operatorname{Ext}(H_{n-1}(C), G) = 0$, then $H^n(C;G) \cong \operatorname{Hom}(H_n(C), G)$.

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