UF MTG 4302/5316 **Practice Exam 2**

Name: ____

- Explain your work (efficiently); partial credit is available.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:
  "I will not give, receive, or use any unauthorized assistance."

Signature: ____

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

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1 Define what it means for a topological space X to be path-connected.

Prove that if X is path-connected, then X is connected.

2 Define what it means for a topological space X to be Hausdorff.

Show that if X is Hausdorff, then the *diagonal* $\Delta = \{(x, x) \mid x \in X\}$ is closed in $X \times X$.

3 Let A and B be topological spaces. Define what it means for a function $q\colon A \to B$ to be a quotient map.

Let $\{X_\alpha\}_{\alpha\in J}$ be a collection of nonempty topological spaces. Let $X = \prod_{\alpha\in J} X_\alpha$. Fix $\beta \in J$. Let $\pi_\beta\colon X \to X_\beta$ be the projection map, defined by $\pi_\beta((x_\alpha)_{\alpha\in J}) = x_\beta$. It is known that π_β is surjective. Show that π_β is a quotient map.

4 Let X be a topological space. Suppose that $\mathcal{C}$ is a collection of closed sets in X having the finite intersection property, i.e., for every finite subcollection $\{C_1, \ldots, C_n\}$ of $\mathcal{C}$, the intersection $C_1 \cap \ldots \cap C_n$ is nonempty. If X is compact, prove that the intersection $\cap_{C \in \mathcal{C}} C$ of all the elements of $\mathcal{C}$ is nonempty.

Remark: I am not asking you to say "This is one direction of a theorem in our book" (the forward direction of Theorem 26.9); I am asking you to prove this.

5 Let (X,d) be a metric space. Suppose X is compact and that $f\colon X \to X$ satisfies $d(f(x),f(y)) = d(x,y)$ for all $x,y \in X$. Show that $f\colon X \to X$ is bijective.

Hint: If $a \notin f(X)$, show there exists some $\varepsilon > 0$ so that the ε-neighborhood of a is disjoint from $f(X)$. Set $x_0 = a$, and $x_{n+1} = f(x_n)$ in general. Show that $d(x_n, x_m) \geq \varepsilon$ for $n > m$.

Remark: It is furthermore true that f is a homeomorphism.

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