UF MTG 6346 **Practice Exam 2**

Name: ______________________________________

- Explain your work (efficiently); partial credit is available.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:
  "I will not give, receive, or use any unauthorized assistance."

Signature: ____

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

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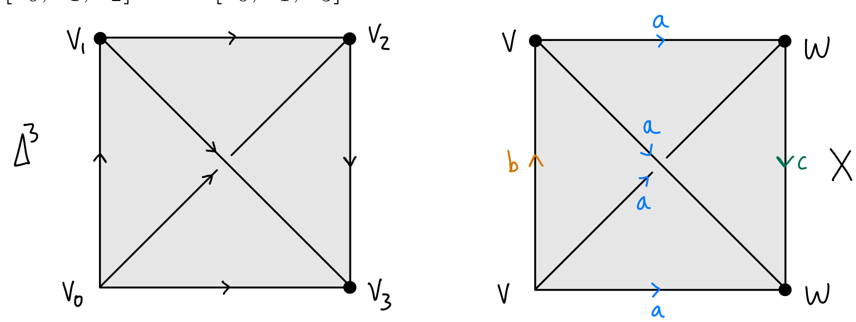
1 Compute the cellular homology of the orientable surface M_g of genus g.

2 Let X be the torus, let $A = X - \{p\}$ be the torus with a single point removed, and let $B \subseteq X$ be a small disc containing the point p. Note $X = \mathrm{int}A \cup \mathrm{int}B$. Derive the 1- and 2-dimensional homology groups of the torus X from the Mayer–Vietoris long exact sequence (and from the known homology groups of A and B).

3 Let X be the Δ-complex obtained from the 3-simplex Δ^3 by identifying $[v_0, v_2, v_3]$ bijectively with $[v_1, v_2, v_3]$ (preserving the ordering of the vertices), and also by similarly identifying $[v_0, v_1, v_2]$ with $[v_0, v_1, v_3]$.



After these identifications, we have two 0-simplices v and w, three 1-simplices a, b, c, two 2-simplices P (the identification of $[v_0, v_2, v_3]$ with $[v_1, v_2, v_3]$) and Q (the identification of $[v_0, v_1, v_2]$ with $[v_0, v_1, v_3]$), and a single 3-simplex T.

Compute the simplicial homology $H_i(X)$ for $i = 0, 1, 2, 3$.

4 Let X and Y be topological spaces. Recall that a continuous map $f\colon X \to Y$ induces a homomorphism $f_\#\colon C_i(X) \to C_i(Y)$ on singular chain groups for all $i \geq 0$. Recall that $f_\#$ commutes with the boundary operators, meaning $f_\# \partial_i^X = \partial_i^Y f_\#$ for all $i \geq 0$.

(a) (3 points) Prove that if $\alpha \in \ker \partial_i^X$, then $f_\#(\alpha) \in \ker \partial_i^Y$.

(b) (3 points) Prove that if $\alpha \in \operatorname{im} \partial_{i+1}^X$, then $f_\#(\alpha) \in \operatorname{im} \partial_{i+1}^Y$.

(c) (4 points) Recall that by (a) and (b) we can define a group homomorphism $f_*\colon H_i(X) \to H_i(Y)$ by $f_*([\alpha]) = [f_\#(\alpha)]$. Suppose now that we have a second continuous map $g\colon X \to Y$. Suppose furthermore that there exists a group homomorphism $P\colon C_i(X) \to C_{i+1}(Y)$ for all $i \geq 0$ that satisfies $P\partial + \partial P = f_\# - g_\#$. Prove that $f_* = g_*\colon H(X) \to H(Y)$.

[5] Just say "True" or "False". No justification is required; no partial credit is available.

(a) Every simplicial complex is also a Δ-complex.

(b) If a chain complex is exact, then all of its homology groups are trivial.

(c) If $0 \to A \to B \to C \to 0$ is a short exact sequence of abelian groups, then $B \cong C/A$.

(d) If a CW complex X has k cells of dimension n, then $H_n(X)$ is generated by at most k elements.

(e) If $-\mathbb{1}\colon S^n \to S^n$ is the antipodal map defined by $-\mathbb{1}(x) = -x$ for all $x \in S^n$, then $\deg(-\mathbb{1}) = -1$.

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