UF MTG 6347 **Practice Exam 2**

Name: ____

- Explain your work (efficiently); partial credit is available.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:
  "I will not give, receive, or use any unauthorized assistance."

Signature: ____

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

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$\boxed{1}$ Prove that $S^2 \vee S^4 \vee S^6$ and $\mathbb{C}P^3$ are not homotopy equivalent.

$\boxed{2}$ Let the closed n-dimensional manifold M have a pair of dual cell structures. Prove the easier version of Poincaré duality that $H^k(M; \mathbb{Z}_2) \cong H_{n-k}(M; \mathbb{Z}_2)$.

$\boxed{3}$ Let M be a closed orientable n-manifold with n odd. Prove that the Euler characteristic $\chi(M)$ is zero.

$\boxed{4}$ For a map $f\colon M \to N$ between connected closed orientable n-manifolds with fundamental classes $[M]$ and $[N]$, the degree of f is defined to be the integer d such that $f_*([M]) = d[N]$, so the sign of the degree depends on the choice of fundamental classes. Show that for any connected closed orientable n-manifold M and any integer d, there is a degree $\pm d$ map $M \to S^n$.

$\boxed{5}$ Just say "True" or "False". No justification is required; no partial credit is available.

(a) For any two free resolutions F and F' of an abelian group A, there are canonical isomorphisms $H^n(F; G) \cong H^n(F'; G)$ for all n.

(b) Any closed manifold M is $\mathbb{Z}_2$-orientable.

(c) For $\alpha \in H^k(X; R)$ and $\beta \in H^l(X; R)$ with R commutative, we have $\alpha \cup \beta = \beta \cup \alpha$.

(d) For $\alpha \in C_{k+l}(X; \mathbb{Z})$, $\varphi \in C^k(X; \mathbb{Z})$, and $\psi \in C^l(X; \mathbb{Z})$, we have

$$(\varphi \cup \psi)(\alpha) = \pm\psi(\alpha \cap \varphi).$$

(e) There exists a closed orientable 10-dimensional manifold M with $H^5(M; \mathbb{Z}) \cong \mathbb{Z}$.

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