UF MTG 4302/5316 **Practice Final**

Name: ______________________________________

- Explain your work (efficiently); partial credit is available.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:
  "I will not give, receive, or use any unauthorized assistance."

Signature: ____

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	50	

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1 (a) Prove that the set of all functions $f\colon \mathbb{Z}_+ \to \{0,1\}$ is uncountable.

(b) We say that a function $f\colon \mathbb{Z}_+ \to \{0,1\}$ is *eventually zero* if there is a positive integer N such that $f(n) = 0$ for all $n \geq N$. Prove that the set of all functions $f\colon \mathbb{Z}_+ \to \{0,1\}$ that are eventually zero is countable.

2 Show that the countable collection $\mathcal{B} = \{(a, b) \mid a, b \in \mathbb{Q} \text{ with } a < b\}$ is a basis that generates the standard topology on $\mathbb{R}$.

(Remark: Since $\mathbb{Q} \times \mathbb{Q}$ is countable, this problem shows that $\mathbb{R}$ has a countable basis for its topology, i.e. $\mathbb{R}$ is second-countable.*)*

3 Let X be a topological space and let $A \subset X$. The *closure of* A is defined by $\overline{A} = \cap\{C \mid C \text{ is closed in } X \text{ and } A \subset C\}$.

(a) Show that $\overline{A}$ is a closed set in X.

(b) Show that $\overline{\overline{A}} = \overline{A}$.

4 (a) Define what it means for a topological space to be *connected*.

(b) Give a precise definition of a topological space that is connected but not path-connected (you do not need to prove your example is correct).

(c) Give a precise definition of a topological space that is connected but not locally connected (you do not need to prove your example is correct).

5 (a) Define the *product topology* on $\mathbb{R}^\omega$.

(b) Define the *box topology* on $\mathbb{R}^\omega$.

(c) Prove that $\mathbb{R}^\omega$ is not connected when equipped with the box topology.
(*Hint: Consider the sets of bounded and unbounded sequences.*)

6 (a) Define what it means for a topological space X to be *Hausdorff*.

(b) Show that if X is Hausdorff and $Y \subset X$ is compact, then Y is closed.

7 Show there is no continuous surjective function $f\colon [0,1] \to [0,1)$.

8 (a) Define what it means for a topological space X to be *normal*.

(b) Let $\mathbb{R}_\ell$ be the real numbers with the lower limit topology, which has as a basis all half-open intervals of the form $[a, b) = \{x \mid a \leq x < b\}$ with $a < b$. Show that $\mathbb{R}_\ell$ is a normal space.

9 Let (X, d) be a metric space.

(a) Define what it means for X to be *complete*.

(b) Suppose there exists some $\varepsilon > 0$ such that for all $x \in X$, the closure $\overline{B_\varepsilon(x)}$ of the ε-ball about x is compact. Prove that X is complete.

10 (a) State the Baire category theorem.

(b) Let X be a Baire space. Prove that if $\{U_n\}$ is a countable collection of dense open sets in X, then $\cap_n U_n$ is also dense in X.

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