UF MTG 4303/5317 **Practice Final**

Name: ________________________________

- Explain your work (efficiently); partial credit is available.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:
  "I will not give, receive, or use any unauthorized assistance."

Signature: ________________________________

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

1. Show that if $h, h' \colon X \to Y$ are homotopic and $k, k' \colon Y \to Z$ are homotopic, then $k \circ h$ and $k' \circ h'$ are homotopic.

2 Let X be a topological space, let $A \subset X$, let $x_0 \in A$, and let $i\colon A \to X$ be the inclusion map. Give an example showing that the induced map $i_*\colon \pi_1(A, x_0) \to \pi_1(X, x_0)$ need not be injective. Prove that if there is a retraction $r\colon X \to A$, then the induced map $i_*\colon \pi_1(A, x_0) \to \pi_1(X, x_0)$ is injective.

3 State the Tietze entension theorem (for maps into the closed interval $[a, b]$, not for maps into $\mathbb{R}$).

4 Let $p\colon E \to B$ be a covering map, and let $p(e_0) = b_0$. Prove that the lifting correspondence $\phi\colon \pi_1(B, b_0) \to p^{-1}(b_0)$ is surjective if E is path connected, and bijective if E is simply connected.

5 Show that a continuous map $f\colon S^1 \to S^1$ is homotopic to $z^n\colon S^1 \to S^1$ (given by $e^{i\theta} \mapsto e^{in\theta}$) for some integer n. [*Hint: Use the covering space* $p\colon \mathbb{R} \to S^1$.]

6 Define what it means for a map $h\colon S^n \to S^m$ to be *antipode-preserving*. Recall that any continuous and antipode-preserving map $h\colon S^1 \to S^1$ is not nullhomotopic. Use this to prove there is no continuous antipode-preserving map $g\colon S^2 \to S^1$.

7 Let S^n be the n-sphere. Prove that $\pi_1(S^n)$ is trivial for $n \geq 2$.

8 Describe a surface whose fundamental group is not abelian.

9 Use the Seifert-van Kampen theorem to compute $\pi_1(\mathbb{R}P^2)$, the fundamental group of the projective plane.

10 Let X be the quotient space obtained from an 8-sided polygonal region P by pasting its edges together according to the labelling scheme $abcdc^{-1}a^{-1}db$. It turns out that all vertices of P are mapped to the same point of the quotient space X by the pasting map. Calculate $H_1(X)$, and using this, determine which compact surface X is homeomorphic to.

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