UF MTG 4303/5317 **Practice Midterm**

Name: ________________________________

- Explain your work (efficiently); partial credit is available.
- No notes, books, calculators, or other electronic devices are permitted.
- Please sign below to indicate you accept the following statement:
  "I will not give, receive, or use any unauthorized assistance."

Signature: ____

Problem	Total Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

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1 Let X be a topological space, and let $f\colon I \to X$ be a loop in X based at $x_0 \in X$. Prove that $f * \overline{f}$ is path homotopic to e_{x_0}.

Remark: Here $e_{x_0}\colon I \to X$ *is the constant loop defined by* $e_{x_0}(s) = x_0$ *for all* $s \in I$.

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2 Let α be a path in X from x_0 to x_1. Define the map $\hat{\alpha}\colon \pi_1(X, x_0) \to \pi_1(X, x_1)$.

Show that $\hat{\alpha}$ is a group isomorphism.

3 Let E and B be topological spaces. Define what it means for a function $p\colon E \to B$ to be a *covering map*.

Prove that if $p\colon E \to B$ and $p'\colon E' \to B'$ are covering maps, then $p \times p'\colon E \times E' \to B \times B'$ defined by $(p \times p')(e, e') = (p(e), p'(e'))$ is a covering map.

4 State the Tietze extension theorem (for maps into the closed interval $[a, b]$, not for maps into $\mathbb{R}$).

The Urysohn lemma states "*If X is a normal space, if A and B are disjoint closed subsets of X, and if $[a, b]$ is a closed interval, then there exists a continuous map $f\colon X \to [a, b]$ such that $f(x) = a$ for all $x \in A$ and $f(x) = b$ for all $x \in B$.*" Use the Tietze extension theorem to prove the Urysohn lemma.

5 Use the fact that there is no continuous antipode-preserving map $g\colon S^2 \to S^1$ in order to prove the Borsuk–Ulam theorem for S^2: "*If $f\colon S^2 \to \mathbb{R}^2$ is continuous, then there is a point $x \in S^2$ with $f(x) = f(-x)$.*"

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