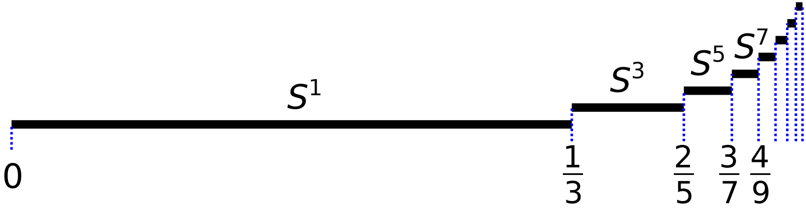
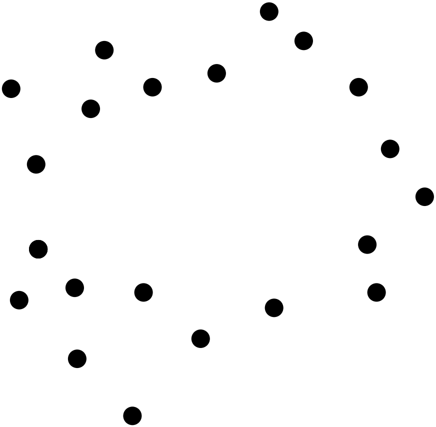
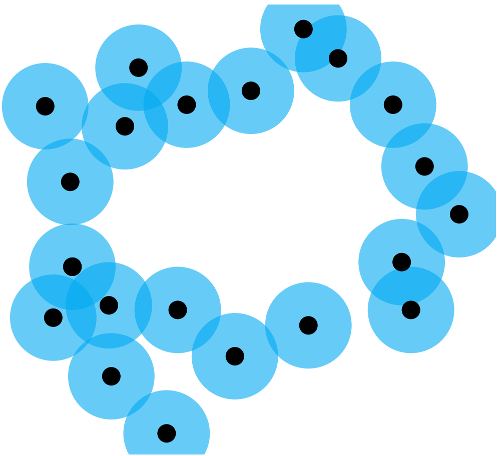
Vietoris–Rips complexes of circles and ellipses

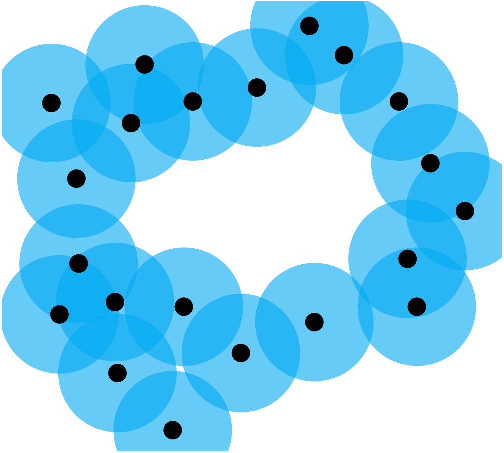
Henry Adams
Colorado State University

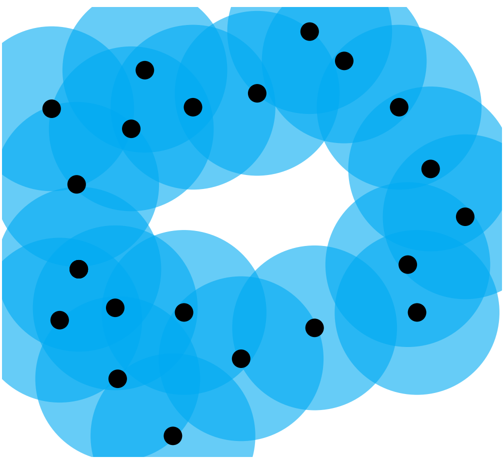
Joint with Michał Adamaszek (University of Copenhagen)
and Samadwara Reddy (Duke University)

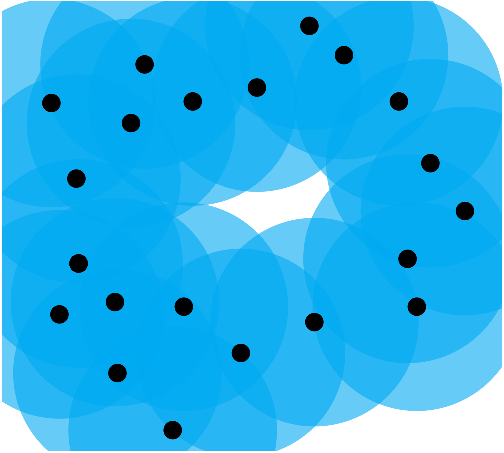


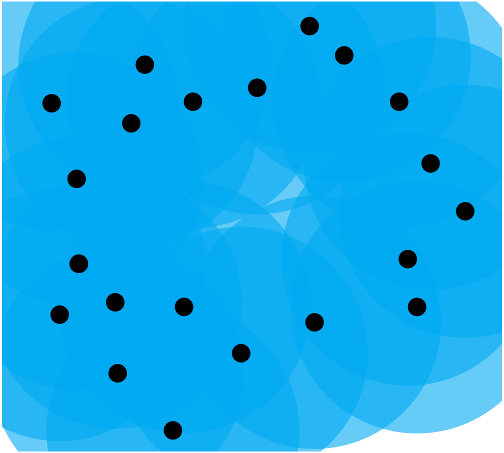


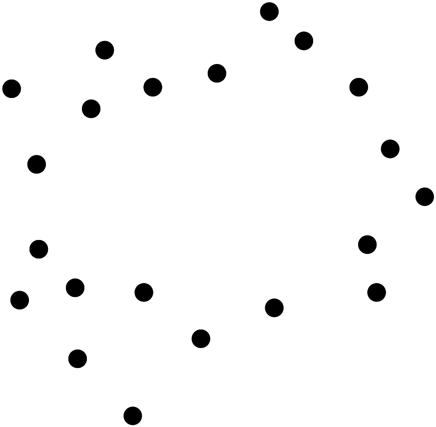








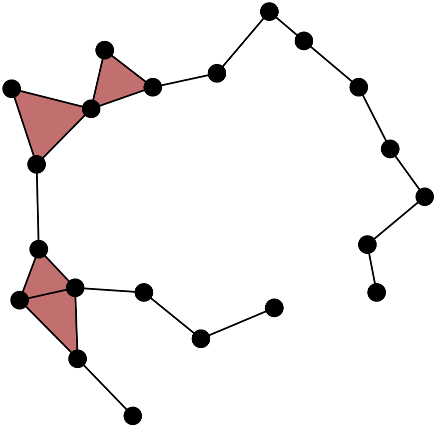




Definition

For metric space X and scale $r \geq 0$, the *Vietoris–Rips simplicial complex* $\mathrm{VR}_{\leq}(X; r)$ has

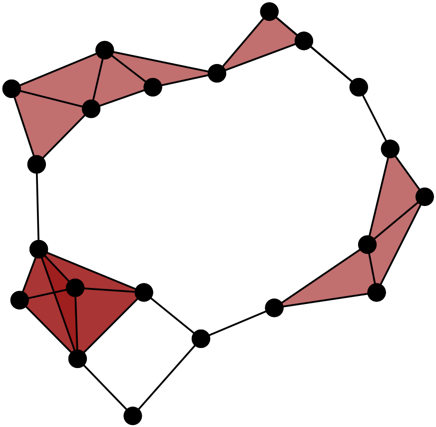
- vertex set X
- finite simplex σ when $\mathrm{diam}(\sigma) \leq r$.



Definition

For metric space X and scale $r \geq 0$, the *Vietoris–Rips simplicial complex* $\mathrm{VR}_{\leq}(X; r)$ has

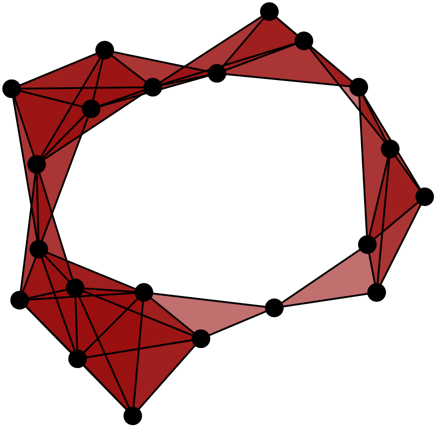
- vertex set X
- finite simplex σ when $\mathrm{diam}(\sigma) \leq r$.



Definition

For metric space X and scale $r \geq 0$, the *Vietoris–Rips simplicial complex* $\mathrm{VR}_{\leq}(X; r)$ has

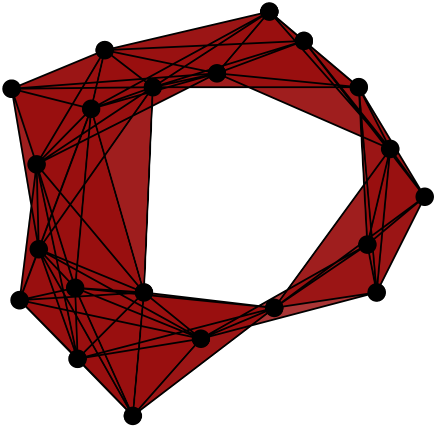
- vertex set X
- finite simplex σ when $\mathrm{diam}(\sigma) \leq r$.



Definition

For metric space X and scale $r \geq 0$, the *Vietoris–Rips simplicial complex* $\mathrm{VR}_{\leq}(X; r)$ has

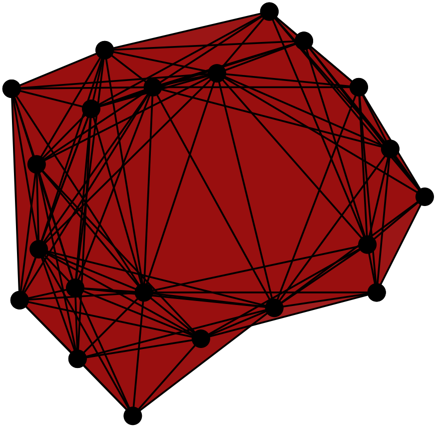
- vertex set X
- finite simplex σ when $\mathrm{diam}(\sigma) \leq r$.



Definition

For metric space X and scale $r \geq 0$, the *Vietoris–Rips simplicial complex* $\mathrm{VR}_{\leq}(X; r)$ has

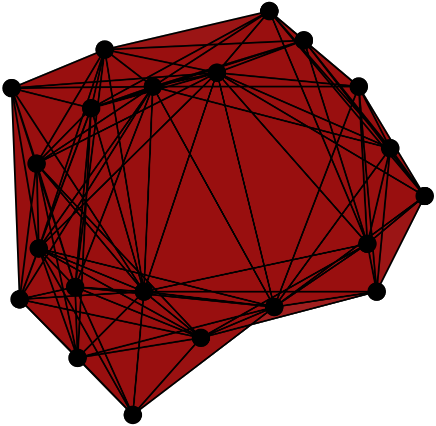
- vertex set X
- finite simplex σ when $\mathrm{diam}(\sigma) \leq r$.



Definition

For metric space X and scale $r \geq 0$, the *Vietoris–Rips simplicial complex* $\mathrm{VR}_{\leq}(X; r)$ has

- vertex set X
- finite simplex σ when $\mathrm{diam}(\sigma) \leq r$.



Definition

For metric space X and scale $r \geq 0$, the *Vietoris–Rips simplicial complex* $\mathrm{VR}_<(X;r)$ has

- vertex set X
- finite simplex σ when $\mathrm{diam}(\sigma) < r$.

Theorem (Hausmann, 1995)

For M a compact Riemannian manifold and r sufficiently small,

$$\mathrm{VR}_<(M;r) \simeq M.$$

Theorem (Hausmann, 1995)

For M a compact Riemannian manifold and r sufficiently small,

$$\mathrm{VR}_<(M;r) \simeq M.$$

Theorem (Latschev, 2001)

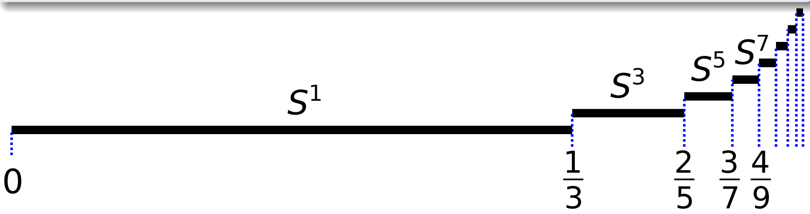
For $X \subseteq M$ sufficiently dense and r sufficiently small,

$$\mathrm{VR}_<(X;r) \simeq M.$$

Let S^1 be the circle of unit circumference.

Theorem (Adamaszek, HA)

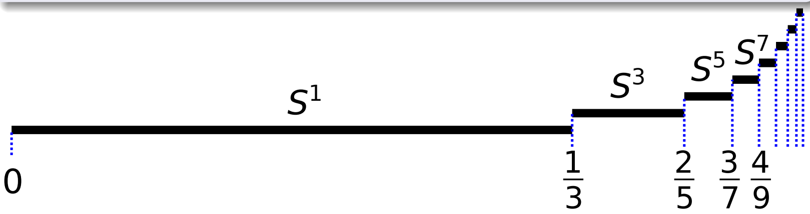
$$\mathrm{VR}_{\leq}(S^1; r) \simeq \begin{cases} S^{2\ell+1} & \text{if } \frac{\ell}{2\ell+1} < r < \frac{\ell+1}{2\ell+3} \quad \text{for some } \ell \in \mathbb{N}. \\ \end{cases}$$



Let S^1 be the circle of unit circumference.

Theorem (Adamaszek, HA)

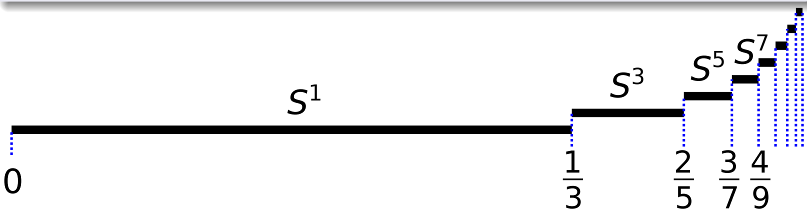
$$\mathrm{VR}_{\leq}(S^1; r) \simeq \begin{cases} S^{2\ell+1} & \text{if } \frac{\ell}{2\ell+1} < r < \frac{\ell+1}{2\ell+3} \\ \bigvee^{\infty} S^{2\ell} & \text{if } r = \frac{\ell}{2\ell+1} \end{cases} \quad \text{for some } \ell \in \mathbb{N}.$$

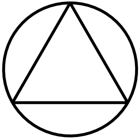
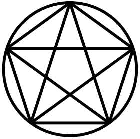
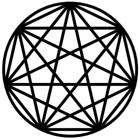


Let S^1 be the circle of unit circumference.

Theorem (Adamaszek, HA)

$$\mathrm{VR}_{\le}(S^1; r) \simeq \begin{cases} S^{2\ell+1} & \text{if } \frac{\ell}{2\ell+1} < r < \frac{\ell+1}{2\ell+3} \\ \bigvee^{\infty} S^{2\ell} & \text{if } r = \frac{\ell}{2\ell+1} \end{cases} \quad \text{for some } \ell \in \mathbb{N}.$$



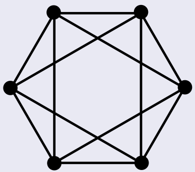
$r = \frac{1}{3}$  $r = \frac{2}{5}$  $r = \frac{3}{7}$ 

Let S^1 be the circle of unit circumference.

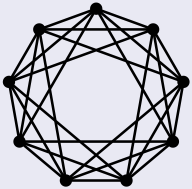
Theorem (Adamaszek, HA)

$$\mathrm{VR}_{\leq}(S^1; r) \simeq \begin{cases} S^{2\ell+1} & \text{if } \frac{\ell}{2\ell+1} < r < \frac{\ell+1}{2\ell+3} \\ \bigvee^{\infty} S^{2\ell} & \text{if } r = \frac{\ell}{2\ell+1} \end{cases} \quad \text{for some } \ell \in \mathbb{N}.$$

Intuition



$\mathrm{VR}_{\leq}(6 \text{ points}, \frac{1}{3}) \simeq S^2$



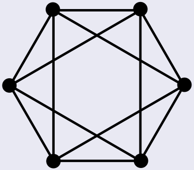
$\mathrm{VR}_{\leq}(9 \text{ points}, \frac{1}{3}) \simeq \bigvee^2 S^2$

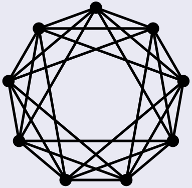
Let S^1 be the circle of unit circumference.

Theorem (Adamaszek, HA)

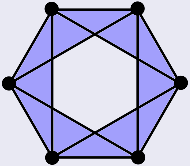
$$\mathrm{VR}_{\le}(S^1; r) \simeq \begin{cases} S^{2\ell+1} & \text{if } \frac{\ell}{2\ell+1} < r < \frac{\ell+1}{2\ell+3} \\ \bigvee^{\infty} S^{2\ell} & \text{if } r = \frac{\ell}{2\ell+1} \end{cases} \quad \text{for some } \ell \in \mathbb{N}.$$

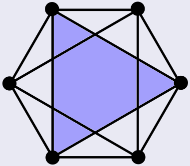
Intuition

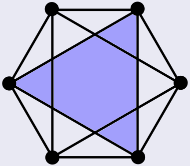




$\mathrm{VR}_{\le}(6 \text{ points}, \frac{1}{3}) \simeq S^2$ $\qquad$ $\mathrm{VR}_{\le}(9 \text{ points}, \frac{1}{3}) \simeq \bigvee^2 S^2$





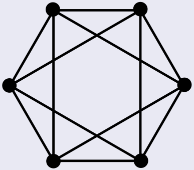


Let S^1 be the circle of unit circumference.

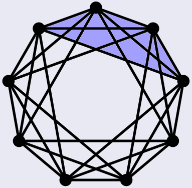
Theorem (Adamaszek, HA)

$$\mathrm{VR}_{\leq}(S^1; r) \simeq \begin{cases} S^{2\ell+1} & \textit{if } \frac{\ell}{2\ell+1} < r < \frac{\ell+1}{2\ell+3} \\ \bigvee^{\infty} S^{2\ell} & \textit{if } r = \frac{\ell}{2\ell+1} \end{cases} \quad \textit{for some } \ell \in \mathbb{N}.$$

Intuition



$\mathrm{VR}_{\leq}(6 \text{ points}, \frac{1}{3}) \simeq S^2$



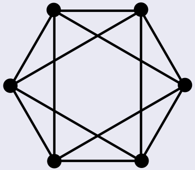
$\mathrm{VR}_{\leq}(9 \text{ points}, \frac{1}{3}) \simeq \bigvee^2 S^2$

Let S^1 be the circle of unit circumference.

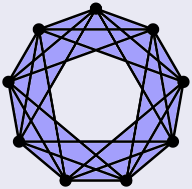
Theorem (Adamaszek, HA)

$$\mathrm{VR}_{\leq}(S^1; r) \simeq \begin{cases} S^{2\ell+1} & \textit{if } \frac{\ell}{2\ell+1} < r < \frac{\ell+1}{2\ell+3} \\ \bigvee^{\infty} S^{2\ell} & \textit{if } r = \frac{\ell}{2\ell+1} \end{cases} \quad \textit{for some } \ell \in \mathbb{N}.$$

Intuition



$\mathrm{VR}_{\leq}(6 \text{ points}, \frac{1}{3}) \simeq S^2$



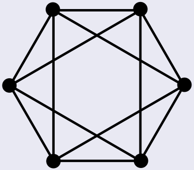
$\mathrm{VR}_{\leq}(9 \text{ points}, \frac{1}{3}) \simeq \bigvee^2 S^2$

Let S^1 be the circle of unit circumference.

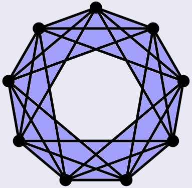
Theorem (Adamaszek, HA)

$$\mathrm{VR}_{\leq}(S^1; r) \simeq \begin{cases} S^{2\ell+1} & \text{if } \frac{\ell}{2\ell+1} < r < \frac{\ell+1}{2\ell+3} \\ \bigvee^{\infty} S^{2\ell} & \text{if } r = \frac{\ell}{2\ell+1} \end{cases} \quad \text{for some } \ell \in \mathbb{N}.$$

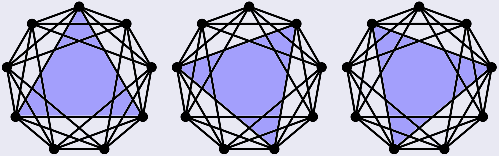
Intuition



$\mathrm{VR}_{\leq}(6 \text{ points}, \frac{1}{3}) \simeq S^2$



$\mathrm{VR}_{\leq}(9 \text{ points}, \frac{1}{3}) \simeq \bigvee^2 S^2$

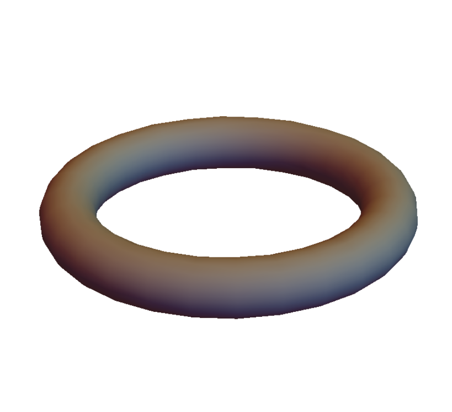


Let S^1 be the circle of unit circumference.

Theorem (Adamaszek, HA)

$$\mathrm{VR}_{\le}(S^1;r) \simeq \begin{cases} S^{2\ell+1} & \text{if } \frac{\ell}{2\ell+1} < r < \frac{\ell+1}{2\ell+3} \\ \bigvee^{\infty} S^{2\ell} & \text{if } r = \frac{\ell}{2\ell+1} \end{cases} \quad \text{for some } \ell \in \mathbb{N}.$$

Intuition

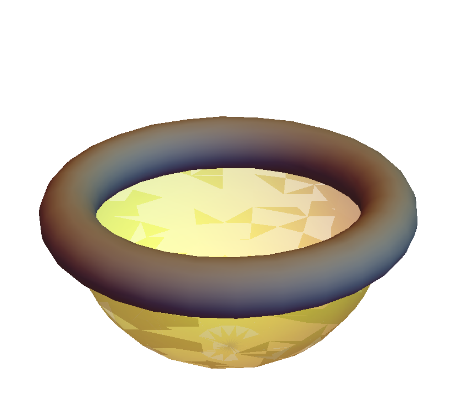


Let S^1 be the circle of unit circumference.

Theorem (Adamaszek, HA)

$$\mathrm{VR}_{\leq}(S^1; r) \simeq \begin{cases} S^{2\ell+1} & \text{if } \frac{\ell}{2\ell+1} < r < \frac{\ell+1}{2\ell+3} \\ \bigvee^{\infty} S^{2\ell} & \text{if } r = \frac{\ell}{2\ell+1} \end{cases} \quad \text{for some } \ell \in \mathbb{N}.$$

Intuition

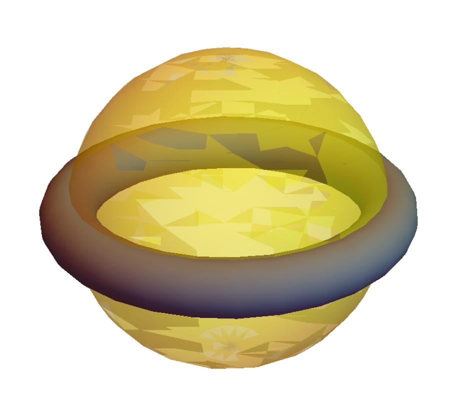


Let S^1 be the circle of unit circumference.

Theorem (Adamaszek, HA)

$$\mathrm{VR}_{\leq}(S^1; r) \simeq \begin{cases} S^{2\ell+1} & \text{if } \frac{\ell}{2\ell+1} < r < \frac{\ell+1}{2\ell+3} \\ \bigvee^{\infty} S^{2\ell} & \text{if } r = \frac{\ell}{2\ell+1} \end{cases} \quad \text{for some } \ell \in \mathbb{N}.$$

Intuition

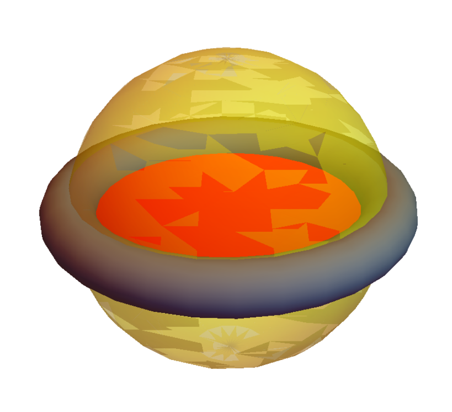


Let S^1 be the circle of unit circumference.

Theorem (Adamaszek, HA)

$$\mathrm{VR}_{\leq}(S^1; r) \simeq \begin{cases} S^{2\ell+1} & \text{if } \frac{\ell}{2\ell+1} < r < \frac{\ell+1}{2\ell+3} \\ \bigvee^{\infty} S^{2\ell} & \text{if } r = \frac{\ell}{2\ell+1} \end{cases} \quad \text{for some } \ell \in \mathbb{N}.$$

Intuition

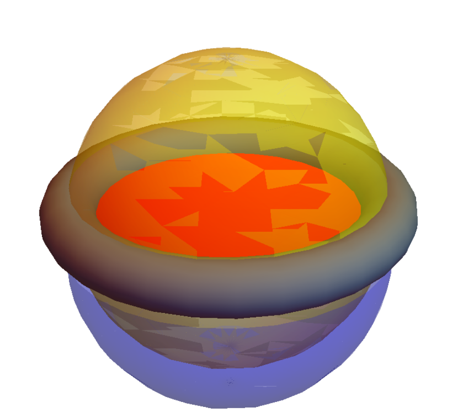


Let S^1 be the circle of unit circumference.

Theorem (Adamaszek, HA)

$$\mathrm{VR}_{\leq}(S^1; r) \simeq \begin{cases} S^{2\ell+1} & \textit{if } \frac{\ell}{2\ell+1} < r < \frac{\ell+1}{2\ell+3} \\ \bigvee^{\infty} S^{2\ell} & \textit{if } r = \frac{\ell}{2\ell+1} \end{cases} \quad \textit{for some } \ell \in \mathbb{N}.$$

Intuition

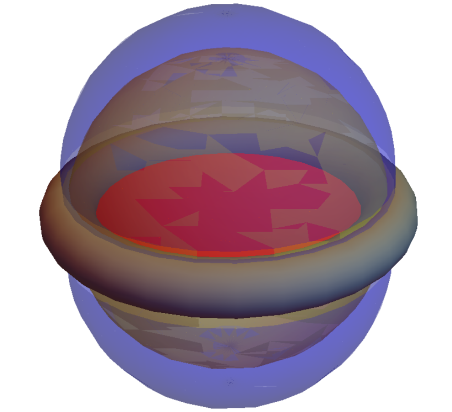


Let S^1 be the circle of unit circumference.

Theorem (Adamaszek, HA)

$$\mathrm{VR}_{\leq}(S^1;r) \simeq \begin{cases} S^{2\ell+1} & \text{if } \frac{\ell}{2\ell+1} < r < \frac{\ell+1}{2\ell+3} \\ \bigvee^{\infty} S^{2\ell} & \text{if } r = \frac{\ell}{2\ell+1} \end{cases} \quad \text{for some } \ell \in \mathbb{N}.$$

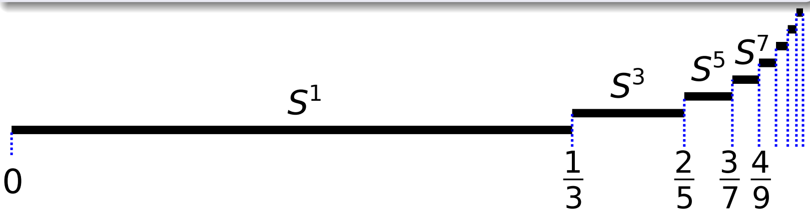
Intuition

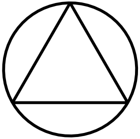
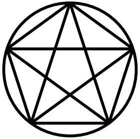
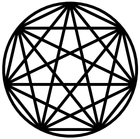


Let S^1 be the circle of unit circumference.

Theorem (Adamaszek, HA)

$$\mathrm{VR}_{\leq}(S^1; r) \simeq \begin{cases} S^{2\ell+1} & \text{if } \frac{\ell}{2\ell+1} < r < \frac{\ell+1}{2\ell+3} \\ \bigvee^{\infty} S^{2\ell} & \text{if } r = \frac{\ell}{2\ell+1} \end{cases} \quad \text{for some } \ell \in \mathbb{N}.$$

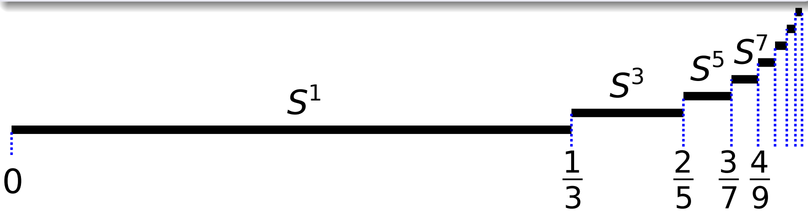


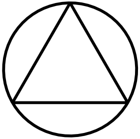
$r = \frac{1}{3}$  $r = \frac{2}{5}$  $r = \frac{3}{7}$ 

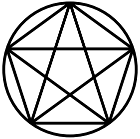
Let S^1 be the circle of unit circumference.

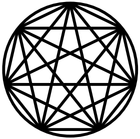
Theorem (Adamaszek, HA)

$$\mathrm{VR}_<(S^1; r) \simeq \begin{cases} S^{2\ell+1} & \text{if } \frac{\ell}{2\ell+1} < r \leq \frac{\ell+1}{2\ell+3} \text{ for some } \ell \in \mathbb{N}. \end{cases}$$



$r = \frac{1}{3}$ 

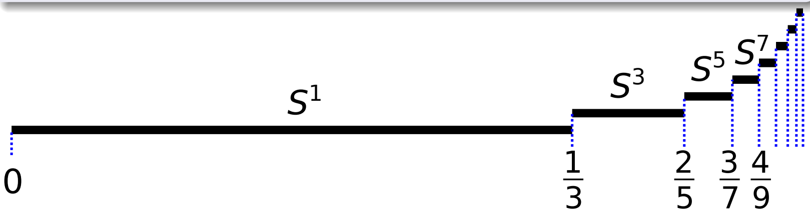
$r = \frac{2}{5}$ 

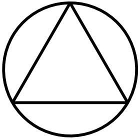
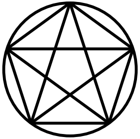
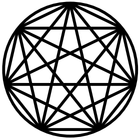
$r = \frac{3}{7}$ 

Let S^1 be the circle of unit circumference.

Theorem (Adamaszek, HA)

$$\mathrm{VR}_<(S^1; r) \simeq \begin{cases} S^{2\ell+1} & \text{if } \frac{\ell}{2\ell+1} < r \leq \frac{\ell+1}{2\ell+3} \quad \text{for some } \ell \in \mathbb{N}. \end{cases}$$



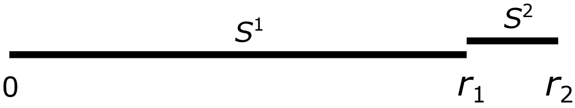
$r = \frac{1}{3}$  $r = \frac{2}{5}$  $r = \frac{3}{7}$ 

Can handle annuli, and tori with the ℓ_∞ metric.

Let $E = \{(x, y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

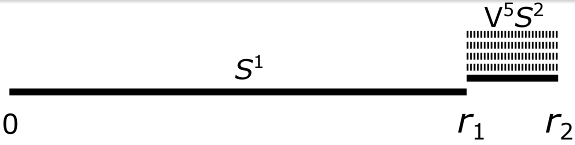
$$\mathrm{VR}_<(E; r) \simeq \begin{cases} S^1 & \text{if } r \leq r_1 \\ S^2 & \text{if } r_1 < r \leq r_2, \end{cases} \quad \text{where } r_1 = \frac{4\sqrt{3}a}{a^2+3}, \; r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$



Let $E = \{(x, y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

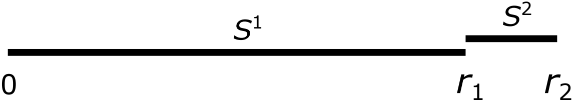
$$\mathrm{VR}_{\leq}(E; r) \simeq \begin{cases} S^1 & \text{if } r < r_1 \\ \bigvee^5 S^2 & \text{if } r_1 < r < r_2, \end{cases} \quad \text{where } r_1 = \frac{4\sqrt{3}a}{a^2+3}, \;\; r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$

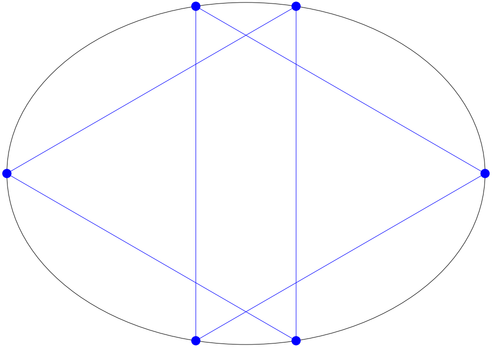


Let $E = \{(x,y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

$$\mathrm{VR}_<(E;r) \simeq \begin{cases} S^1 & \text{if } r \leq r_1 \\ S^2 & \text{if } r_1 < r \leq r_2, \end{cases} \quad \text{where } r_1 = \frac{4\sqrt{3}a}{a^2+3},\ \ r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$

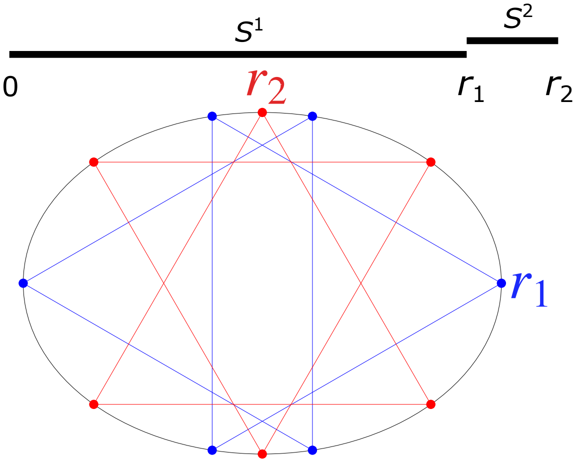




Let $E = \{(x, y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

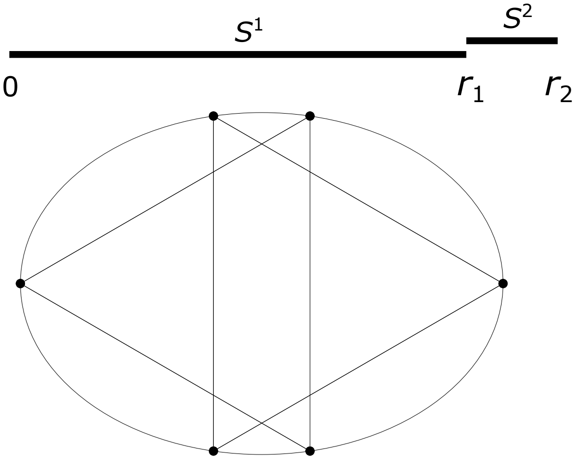
$$\mathrm{VR}_<(E; r) \simeq \begin{cases} S^1 & \text{if } r \leq r_1 \\ S^2 & \text{if } r_1 < r \leq r_2, \end{cases} \quad \text{where } r_1 = \frac{4\sqrt{3}a}{a^2+3}, \; r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$



Let $E = \{(x,y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

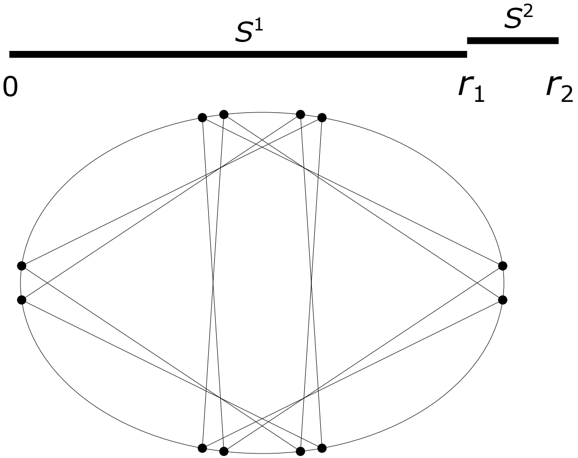
$$\mathrm{VR}_<(E;r) \simeq \begin{cases} S^1 & \text{if } r \leq r_1 \\ S^2 & \text{if } r_1 < r \leq r_2, \end{cases} \quad \text{where } r_1 = \frac{4\sqrt{3}a}{a^2+3},\ r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$



Let $E = \{(x, y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

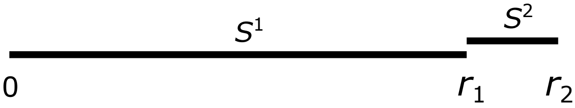
$$\mathrm{VR}_<(E; r) \simeq \begin{cases} S^1 & \text{if } r \leq r_1 \\ S^2 & \text{if } r_1 < r \leq r_2, \end{cases} \quad \textit{where } r_1 = \frac{4\sqrt{3}a}{a^2+3}, \;\; r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$

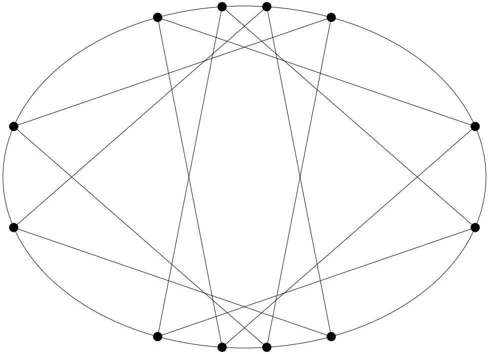


Let $E = \{(x, y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

$$\mathrm{VR}_<(E; r) \simeq \begin{cases} S^1 & \text{if } r \leq r_1 \\ S^2 & \text{if } r_1 < r \leq r_2, \end{cases} \quad \text{where } r_1 = \frac{4\sqrt{3}a}{a^2+3}, \ \ r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$

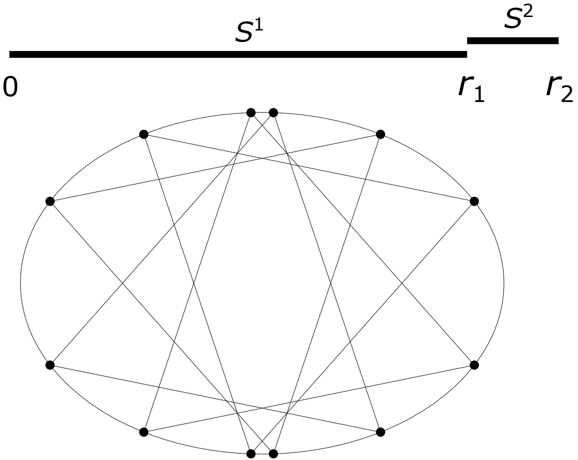




Let $E = \{(x, y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

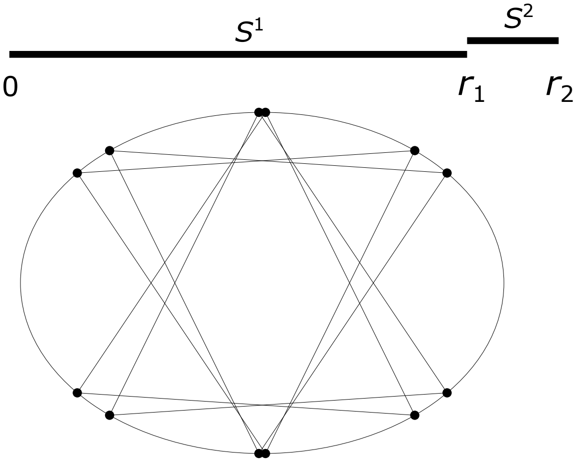
$$\mathrm{VR}_<(E; r) \simeq \begin{cases} S^1 & \text{if } r \leq r_1 \\ S^2 & \text{if } r_1 < r \leq r_2, \end{cases} \quad \text{where } r_1 = \frac{4\sqrt{3}a}{a^2+3}, \ \ r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$



Let $E = \{(x, y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

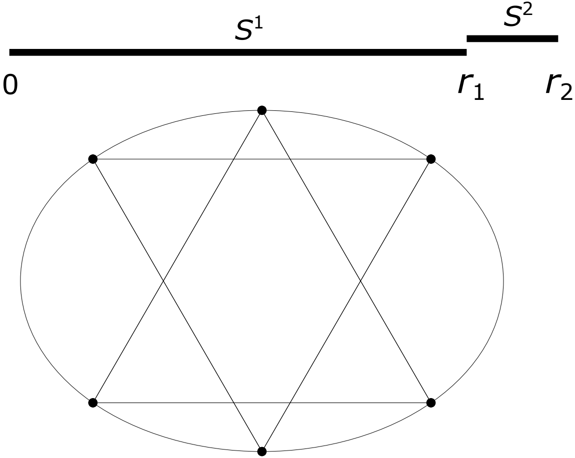
$$\mathrm{VR}_<(E; r) \simeq \begin{cases} S^1 & \text{if } r \leq r_1 \\ S^2 & \text{if } r_1 < r \leq r_2, \end{cases} \quad \text{where } r_1 = \frac{4\sqrt{3}a}{a^2+3}, \;\; r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$



Let $E = \{(x,y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

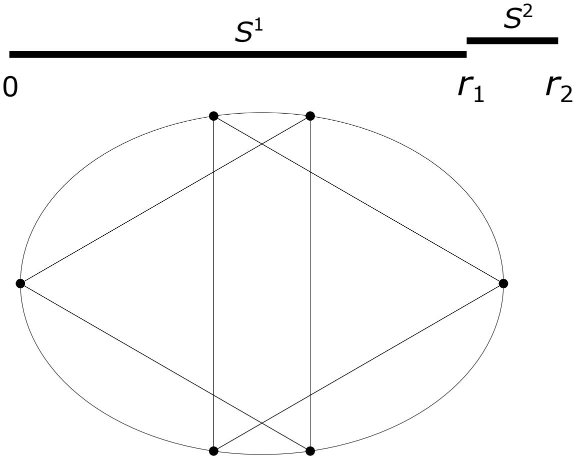
$$\mathrm{VR}_<(E;r) \simeq \begin{cases} S^1 & \text{if } r \leq r_1 \\ S^2 & \text{if } r_1 < r \leq r_2, \end{cases} \quad \text{where } r_1 = \frac{4\sqrt{3}a}{a^2+3},\ \ r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$



Let $E = \{(x, y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

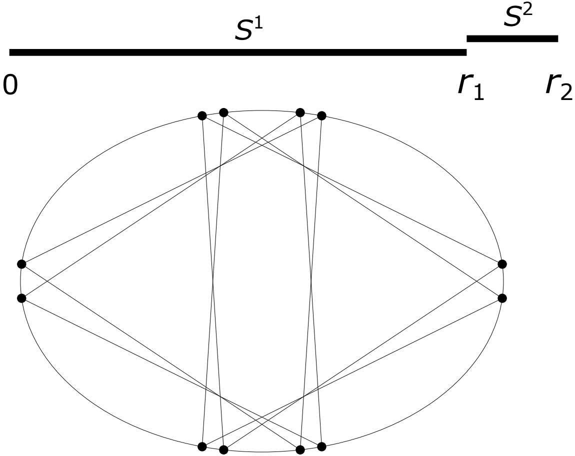
$$\mathrm{VR}_{<}(E; r) \simeq \begin{cases} S^1 & \text{if } r \leq r_1 \\ S^2 & \text{if } r_1 < r \leq r_2, \end{cases} \quad \textit{where } r_1 = \frac{4\sqrt{3}a}{a^2+3}, \;\; r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$



Let $E = \{(x, y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

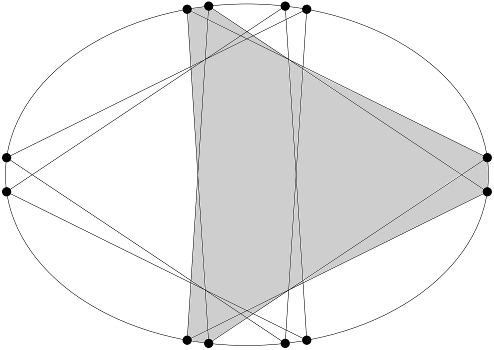
$$\mathrm{VR}_<(E; r) \simeq \begin{cases} S^1 & \text{if } r \leq r_1 \\ S^2 & \text{if } r_1 < r \leq r_2, \end{cases} \quad \text{where } r_1 = \frac{4\sqrt{3}a}{a^2+3}, \;\; r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$



Let $E = \{(x, y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

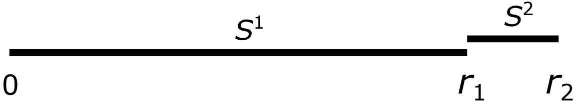
$$\mathrm{VR}_<(E; r) \simeq \begin{cases} S^1 & \text{if } r \leq r_1 \\ S^2 & \text{if } r_1 < r \leq r_2, \end{cases} \quad \text{where } r_1 = \frac{4\sqrt{3}a}{a^2+3},\ \ r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$

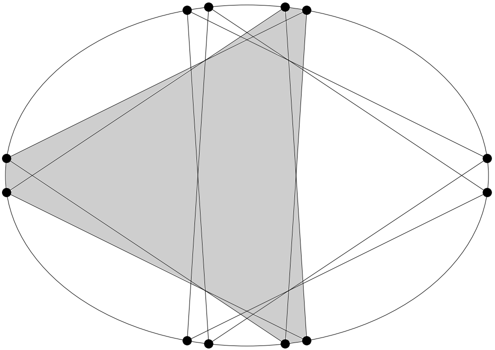


Let $E = \{(x, y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

$$\mathrm{VR}_{<}(E; r) \simeq \begin{cases} S^1 & \text{if } r \leq r_1 \\ S^2 & \text{if } r_1 < r \leq r_2, \end{cases} \quad \text{where } r_1 = \frac{4\sqrt{3}a}{a^2+3}, \; r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$

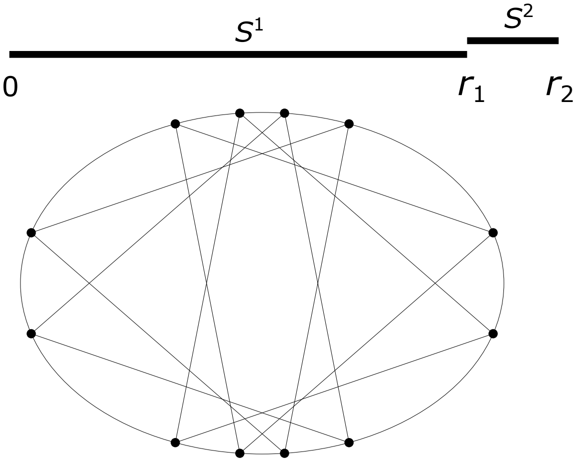




Let $E = \{(x, y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \le \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

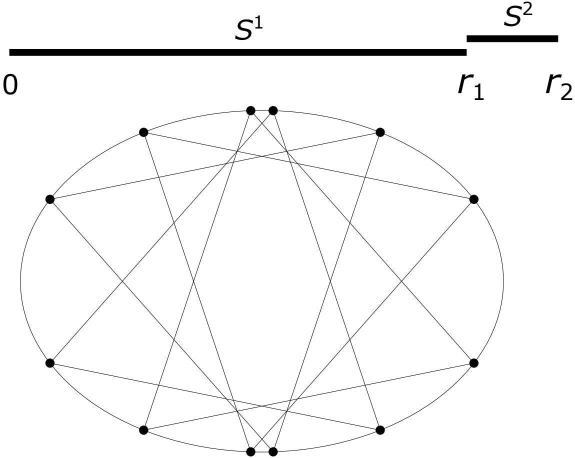
$$\mathrm{VR}_<(E; r) \simeq \begin{cases} S^1 & \text{if } r \le r_1 \\ S^2 & \text{if } r_1 < r \le r_2, \end{cases} \quad \text{where } r_1 = \frac{4\sqrt{3}a}{a^2+3}, \;\; r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$



Let $E = \{(x,y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

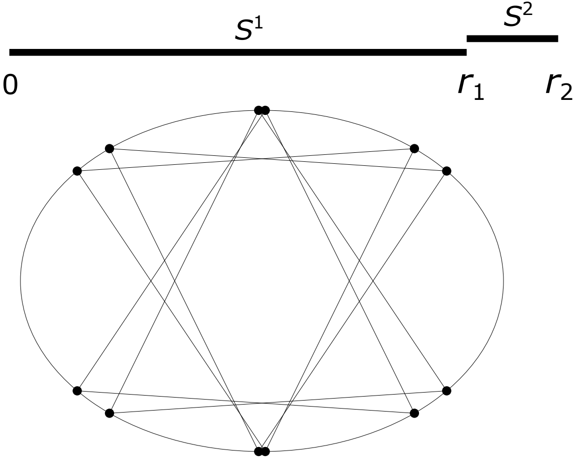
$$\mathrm{VR}_<(E;r) \simeq \begin{cases} S^1 & \text{if } r \leq r_1 \\ S^2 & \text{if } r_1 < r \leq r_2, \end{cases} \quad \text{where } r_1 = \frac{4\sqrt{3}a}{a^2+3}, \; r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$



Let $E = \{(x, y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

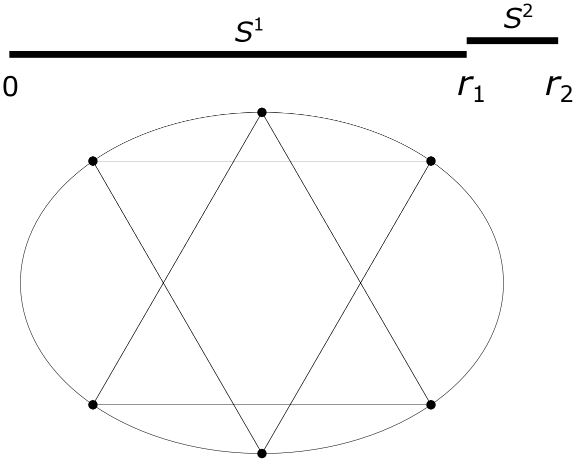
$$\mathrm{VR}_{<}(E; r) \simeq \begin{cases} S^1 & \text{if } r \leq r_1 \\ S^2 & \text{if } r_1 < r \leq r_2, \end{cases} \quad \text{where } r_1 = \frac{4\sqrt{3}a}{a^2+3}, \ r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$



Let $E = \{(x,y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

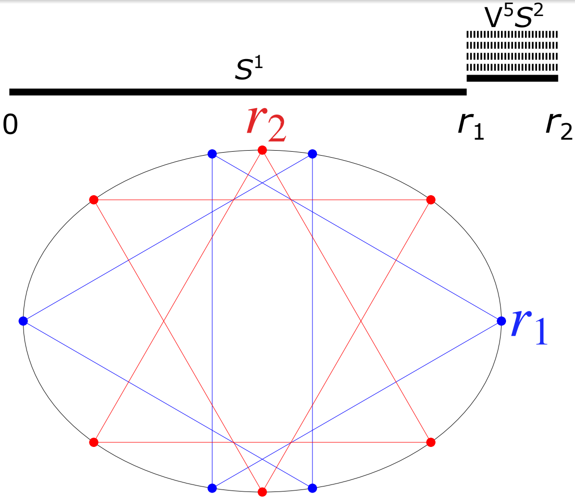
$$\mathrm{VR}_<(E;r) \simeq \begin{cases} S^1 & \text{if } r \leq r_1 \\ S^2 & \text{if } r_1 < r \leq r_2, \end{cases} \quad \text{where } r_1 = \frac{4\sqrt{3}a}{a^2+3}, \ \ r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$



Let $E = \{(x, y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

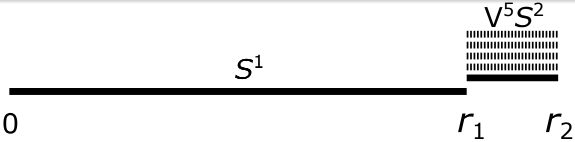
$$\mathrm{VR}_{\leq}(E; r) \simeq \begin{cases} S^1 & \text{if } r < r_1 \\ \bigvee^5 S^2 & \text{if } r_1 < r < r_2, \end{cases} \quad \text{where } r_1 = \frac{4\sqrt{3}a}{a^2+3},\ \ r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$

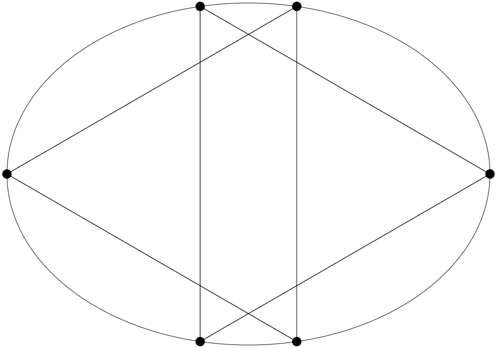


Let $E = \{(x,y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

$$\mathrm{VR}_{\leq}(E;r) \simeq \begin{cases} S^1 & \text{if } r < r_1 \\ \bigvee^5 S^2 & \text{if } r_1 < r < r_2, \end{cases} \quad \text{where } r_1 = \frac{4\sqrt{3}a}{a^2+3},\ \ r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$

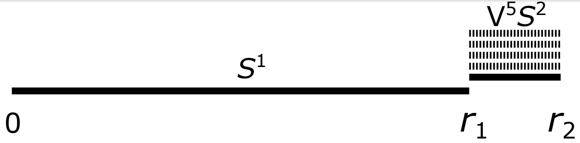


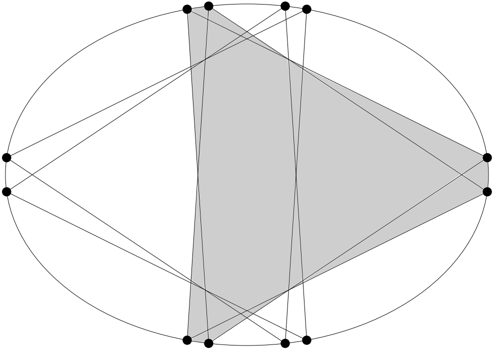


Let $E = \{(x, y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \le \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

$$\mathrm{VR}_{\le}(E; r) \simeq \begin{cases} S^1 & \textit{if } r < r_1 \\ \bigvee^5 S^2 & \textit{if } r_1 < r < r_2, \end{cases} \quad \textit{where } r_1 = \frac{4\sqrt{3}a}{a^2+3}, \ r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$

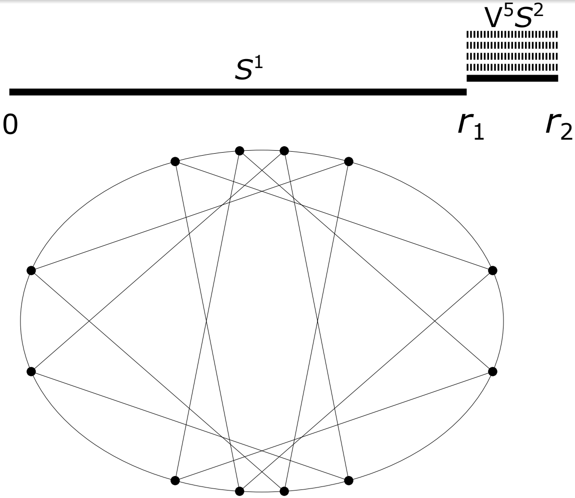




Let $E = \{(x,y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

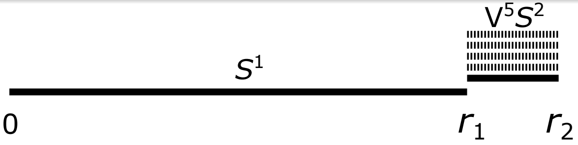
$$\mathrm{VR}_{\leq}(E;r) \simeq \begin{cases} S^1 & \text{if } r < r_1 \\ \bigvee^5 S^2 & \text{if } r_1 < r < r_2, \end{cases} \quad \text{where } r_1 = \frac{4\sqrt{3}a}{a^2+3},\ \ r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$

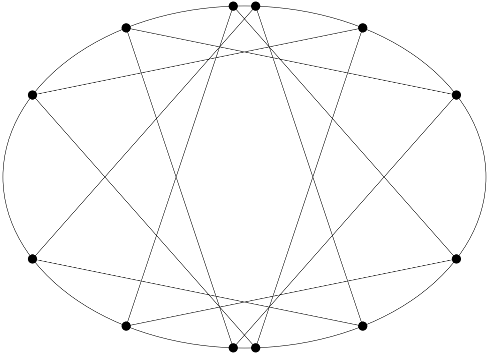


Let $E = \{(x, y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

$$\mathrm{VR}_{\leq}(E; r) \simeq \begin{cases} S^1 & \text{if } r < r_1 \\ \bigvee^5 S^2 & \text{if } r_1 < r < r_2, \end{cases} \quad \text{where } r_1 = \frac{4\sqrt{3}a}{a^2+3}, \;\; r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$

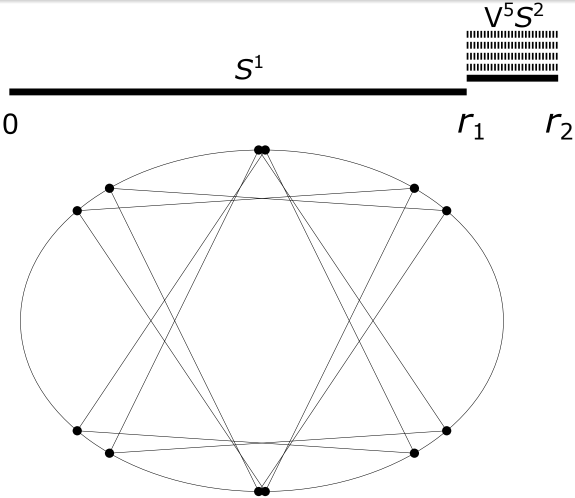




Let $E = \{(x, y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

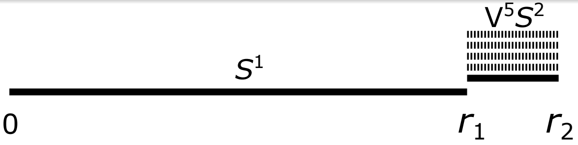
$$\mathrm{VR}_{\leq}(E; r) \simeq \begin{cases} S^1 & \text{if } r < r_1 \\ \bigvee^5 S^2 & \text{if } r_1 < r < r_2, \end{cases} \quad \text{where } r_1 = \frac{4\sqrt{3}a}{a^2+3}, \ r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$

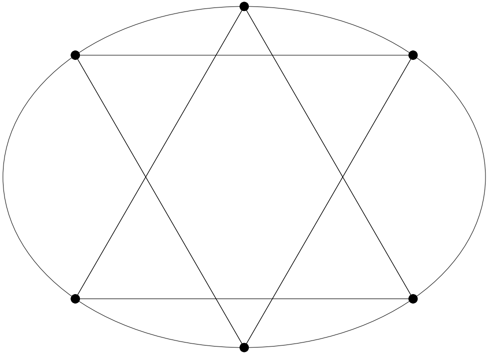


Let $E = \{(x, y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

$$\mathrm{VR}_{\leq}(E; r) \simeq \begin{cases} S^1 & \text{if } r < r_1 \\ \bigvee^5 S^2 & \text{if } r_1 < r < r_2, \end{cases} \quad \text{where } r_1 = \frac{4\sqrt{3}a}{a^2+3}, \;\; r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$

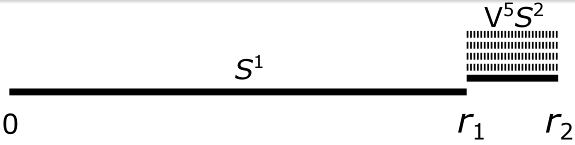




Let $E = \{(x,y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

$$\mathrm{VR}_{\leq}(E;r) \simeq \begin{cases} S^1 & \text{if } r < r_1 \\ \bigvee^5 S^2 & \text{if } r_1 < r < r_2, \end{cases} \quad \text{where } r_1 = \frac{4\sqrt{3}a}{a^2+3},\ \ r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$



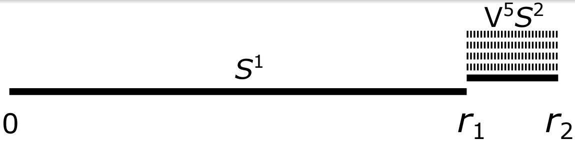
Theorem (Adamaszek, HA, Reddy)

For any $r_1 < r < r_2$, $\epsilon > 0$, and $i \geq 1$, there exists an ϵ-dense finite subset $X \subseteq E$ with $\mathrm{VR}_{<}(X;r) \simeq \bigvee^i S^2$.

Let $E = \{(x, y) \mid (x/a)^2 + y^2 = 1\}$ be an ellipse with $1 < a \leq \sqrt{2}$ (Euclidean metric).

Theorem (Adamaszek, HA, Reddy)

$$\mathrm{VR}_{\leq}(E; r) \simeq \begin{cases} S^1 & \textit{if } r < r_1 \\ \bigvee^5 S^2 & \textit{if } r_1 < r < r_2, \end{cases} \quad \textit{where } r_1 = \frac{4\sqrt{3}a}{a^2+3}, \ \ r_2 = \frac{4\sqrt{3}a^2}{3a^2+1}.$$



Future work

1. $\mathrm{VR}_{<}(E; r)$ for larger r.
2. $\mathrm{VR}_{<}(S^n; r)$?
3. For M a Riemannian manifold do we have $\mathrm{VR}_{<}(X_{\text{suff. dense}}; r) \xrightarrow{\simeq} \mathrm{VR}_{<}(M; r)$?
   False for $M = E$ (not Riemannian).
4. Is $\mathrm{conn}(\mathrm{VR}_{<}(M; r))$ a non-decreasing function of r?
5. Ambient Čech complexes.

References

- Michał Adamaszek and Henry Adams, *The Vietoris–Rips complexes of a circle*, arXiv:1503.03669 (2015).
- Michał Adamaszek, Henry Adams, and Francis Motta, *Random cyclic dynamical systems*, arXiv:1511.07832 (2015).
- Michał Adamaszek, Henry Adams, Florian Frick, Chris Peterson, and Corrine Previte–Johnson, *Nerve and clique complexes of circular arcs*, arXiv:1410.4336 (2014).
- Michał Adamaszek, *Clique complexes and graph powers*, Israel Journal of Mathematics 196 (2013), 295–319.
- Frédéric Chazal, William Crawley-Boevey, Vin de Silva, *The observable structure of persistence modules*, arXiv:1405.5644 (2014).
- Jean-Claude Hausmann, *On the Vietoris–Rips complexes and a cohomology theory for metric spaces*, Annals of Mathematics Studies 138 (1995), 175–188.
- Janko Latschev, *Vietoris–Rips complexes of metric spaces near a closed Riemannian manifold*, Archiv der Mathematik 77 (2001), 522–528.

Thank you!

Theorem (Hausman, 1995)

Let M be a Riemannian manifold with $r(M) > 0$.
If $0 < r \leq r(M)$, then $\mathrm{VR}_{\leq}(M; r) \simeq M$.

Definition

Let $r(M)$ be the largest satisfying:
(a) If $d(x, y) < 2r(M)$, then $\exists!$ shortest geodesic between x and y.

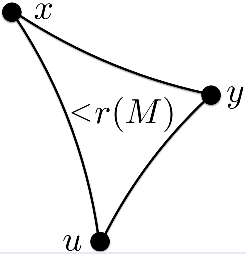
Theorem (Hausman, 1995)

Let M be a Riemannian manifold with $r(M) > 0$.
If $0 < r \leq r(M)$, then $\mathrm{VR}_{\leq}(M; r) \simeq M$.

Definition

Let $r(M)$ be the largest satisfying:

(a) If $d(x, y) < 2r(M)$, then $\exists!$ shortest geodesic between x and y.

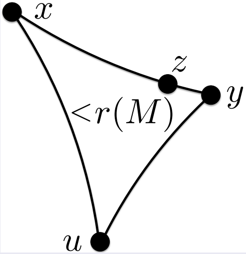
(b) 

Theorem (Hausman, 1995)

Let M be a Riemannian manifold with $r(M) > 0$.
If $0 < r \leq r(M)$, then $\mathrm{VR}_{\leq}(M; r) \simeq M$.

Definition

Let $r(M)$ be the largest satisfying:
(a) If $d(x, y) < 2r(M)$, then $\exists!$ shortest geodesic between x and y.

(b) 

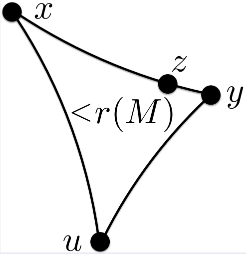
Theorem (Hausman, 1995)

Let M be a Riemannian manifold with $r(M) > 0$.
If $0 < r \leq r(M)$, then $\mathrm{VR}_{\leq}(M; r) \simeq M$.

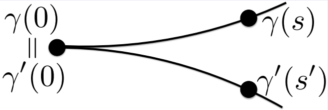
Definition

Let $r(M)$ be the largest satisfying:
(a) If $d(x, y) < 2r(M)$, then $\exists!$ shortest geodesic between x and y.

(b)



(c)



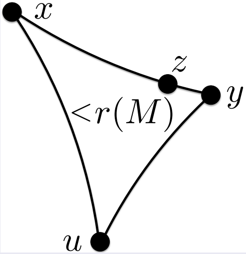
Theorem (Hausman, 1995)

Let M be a Riemannian manifold with $r(M) > 0$.
If $0 < r \leq r(M)$, then $\mathrm{VR}_{\leq}(M; r) \simeq M$.

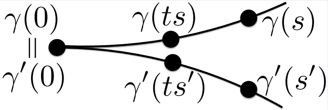
Definition

Let $r(M)$ be the largest satisfying:
(a) If $d(x, y) < 2r(M)$, then $\exists!$ shortest geodesic between x and y.

(b)



(c)

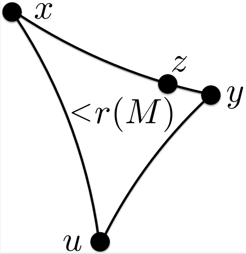


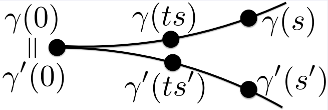
Theorem (Hausman, 1995)

Let M be a Riemannian manifold with $r(M) > 0$.
If $0 < r \leq r(M)$, then $\mathrm{VR}_{\leq}(M; r) \simeq M$.

Definition

Let $r(M)$ be the largest satisfying:
(a) If $d(x, y) < 2r(M)$, then $\exists!$ shortest geodesic between x and y.

(b) 

(c) 

- The n-sphere with great circle circumference 1 has $r(S^n) = \frac{1}{4}$.
- $r(M) > 0$ if M has positive injectivity radius and bounded sectional curvature (in particular if M compact).