

2. DE BRUIJN SEQUENCES

Definition 2.1. A binary *de Bruijn sequence* of order n is a sequence of 0's and 1's such that when arranged around a circle every length- n sequence of 0's and 1's appears exactly once.

Variants: having order n is sometimes called having “window n ”. Do you see why? You can also use more symbols than just 0's and 1's, in which case the sequence is no longer binary.

(4) Find a binary de Bruijn sequence of order 2.

(5) Find a binary de Bruijn sequence of order 4.

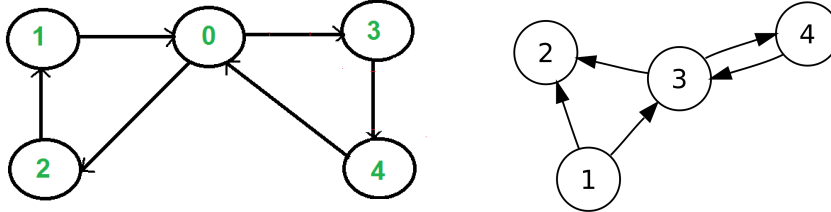
(6) Are there binary de Bruijn sequences of order n for all n ?

(7) (Digression) Can you find a de Bruijn sequence of order 3 using the numbers 0, 1, and 2? How long must it be?

3. EULERIAN CIRCUITS

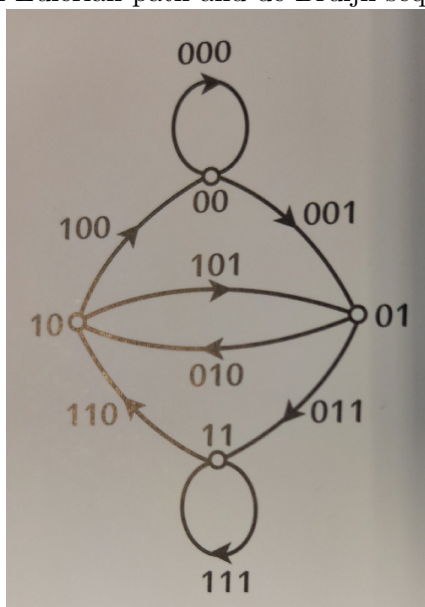
Definition 3.1. An Eulerian circuit is a path which starts and ends at the same node and uses each edge exactly once.

(8) Find Eulerian circuits in the following, or explain why it is not possible:



(9) Can you always find an Eulerian circuits if each node has an equal number of edges coming in and out?

(10) Why is the following graph called a de Bruijn graph, and what is the connection between an Eulerian path and de Bruijn sequences?



4. CONCLUSION

(11) Use de Bruijn graphs and Eulerian circuits to give systematic constructions of de Bruijn sequences of order 2, 4, and 5.

(12) Can you explain the magic trick using de Bruijn sequences? Can you perform the trick yourself?

(13) How might de Bruijn sequences help a robot navigate in a lab?

(14) Can you construct de Bruijn sequences using three or more symbols by generalizing the de Bruijn graph?

If you want to see more mathematical magic tricks, I recommend “Magical Mathematics: The Mathematical Ideas that Animate Great Magic Tricks” by Persi Diaconis and Ron Graham.