

THE ISLAND OF KNIGHTS AND LIARS

UNIVERSITY OF FLORIDA MATH CIRCLE

1. ISLAND OF KNIGHTS AND LIARS

Knights and liars are logically minded people. Knights always tell the truth, and liars always lie whenever they say anything. You're visiting an island inhabited by only knights and liars, but you don't know who is who! However, with carefully asked questions to everyone you meet, you will be able to figure out who are knights and who are liars.¹

(1) You meet someone on the island and immediately ask whether they are a knight. How do they answer?

(2) Realizing your mistake, you think for a little bit. Then you ask, "If I asked if you were a knight, what would you answer?". They reply "no". What are they?²

(3) On the island of knights and liars, you are approached by two people. The first one says to you, "we are both liars." What are they actually?

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¹These problems are adapted from sessions of the Tucson and UCLA math circles and the vast store of such puzzles on the internet. Sometimes liars are called knaves.

²If you find the conditional "if I asked you..." confusing, an equivalent formulation is to ask "Are you a knight" and have them write the answer down. Then ask them what they wrote down.

(4) Dana and Sergei approach you on the island. Dana says “Sergei is a liar.” Sergei says “Neither of us are liars.” What are they?

(5) Walking along, you meet Betsy and Miriam on the island, and begin to have a conversation. Realizing they could be lying about everything, you finally ask about whether they are knights or liars. In response, Betsy says, “If you asked Miriam if she was a knight, she would say no,” and Miriam says, “If you asked Betsy if she was a knight, she would say yes.” What are they?

(6) On the island of knights and liars, you come to a fork in the road with one man standing before each path. You know that one of them is a knight, and the other is a liar. You also know that one path leads to safety, and the other path leads to certain death. You can ask one of the men one question. What do you ask to determine the path to safety?

- (7) Afterward escaping to safety, you meet Hector and Miguel, and ask them whether they are knights or liars. Hector says, "If you asked Miguel if I was a knight, he would say no." Miguel says, "If you asked Hector if I was a knight, he would say yes." What are they?
- (8) Now you come across another pair of islanders, one named Marta and one named Jose. You ask Marta, "Is Jose a liar?" Rather than respond to you, Marta whispers the answer to Jose. You ask Jose whether Marta whispered "Yes", and Jose replies, "No!" What is Marta?
- (9) While talking with a local knight, I asked her the same question twice, and she gave me two different answers. What could be my question?

2. ADDITIONAL ISLANDER QUESTIONS

- (1) In a group of 10 islanders, the first one says “Exactly one of us is a liar.” The second one says “Exactly two of us are liars.” The pattern continues until the tenth one says “Exactly ten of us are liars.” How many of them are liars?
- (2) In a group of 10 islanders, the first one says “At least one of us is a liar.” The second one says “At least two of us are liars.” The pattern continues until the tenth one says “At least ten of us are liars.” How many of them are liars?
- (3) A group of ten islanders stands in a line. The first five in the line say “Everyone behind me is a liar.” The last five in the line say “Everyone in front of me is a liar.” How many liars are there?

- (4) A group of thirty islanders stand in a circle. Everyone says “The people to my right and left are both liars.” What is the maximum possible number of liars?

- (5) Create a knight and liar puzzle of your own. Make sure there is only one answer, and then be prepared to share it.

3. ADDITIONAL ISLANDER QUESTIONS (NEW COMPLICATIONS)

- (1) I met two men who lived on the island and asked the taller man “Are both of you Knights?”. He replied with a “Yes” or “No”, but from his answer, I could not figure out what type of person each man was. I then asked the shorter man “Is the taller man a Knight?”. He replied with a “Yes” or “No”, and after that I knew which type of person each man was. What were they?
- (2) You meet three inhabitants: Alice, Evelyn and Bob. Alice tells you that Evelyn is a liar. Evelyn tells you that Bob isn’t a liar. Bob claims, “I am a knight or Alice is a knight.” What can you say about Alice, Evelyn, and Bob?
- (3) You approach another fork in the road. A person stands before you, either a knight or a liar.
You know that one road leads to freedom, and the other to certain death. You may ask the person one questions, to which they will respond either “ja” or “da.” How do you determine the path to freedom?
- (4) You have met a group of 6 islanders who are either knights or liars. Their names are Justin, Robert, Eustice, Uri, Arthur, and Beatrix.
Justin says: Robert never tells the truth.
Eustice says: Justin never tells the truth.
Robert says: Eustice never lies.
Beatrix says: Uri tells the truth.
Arthur says: Beatrix always lies.
Uri says: Arthur is untruthful.
Justin says: Uri is a liar and I am a liar.
Who is a liar and who is a knight?

Tourists also visit the island. Tourists sometimes lie and sometimes tell the truth.

- (5) Jay says that he is a liar. Can you figure out whether he is a knight, liar or tourist? Tracy says that she is a tourist. Can you figure out what she is?
- (6) Matt, Tristan, and Katie are having dinner. You know one is a knight, one is a liar, and one is a tourist. Matt says that he is a knight. Tristan says “Matt speaks the truth.” Katie says that she is a tourist. What are they?
- (7) On the island, you come across three men. You know that one is a knight, one is a liar, and one is a tourist - but you don’t know who’s who.
You are allowed to ask these three gentlemen two yes-or-no questions. They will all answer you, one at a time, with either a “yes” or a “no.” How do you determine the true identities of these fellows?
- (8) On the island of knights and liars, you come to a fork in the road. One path leads to death, the other path leads you to a boat so you can escape this perplexing island. At the intersection, you encounter three women. You know one is a knight, one is a liar, and one is a tourist – but you do not know who is who.
You may ask two questions. Each question must be addressed to only one woman, and only she will respond. How do you escape the island?