

UF MATH CIRCLE: SEPTEMBER 15, 2022

1. SPROUTS

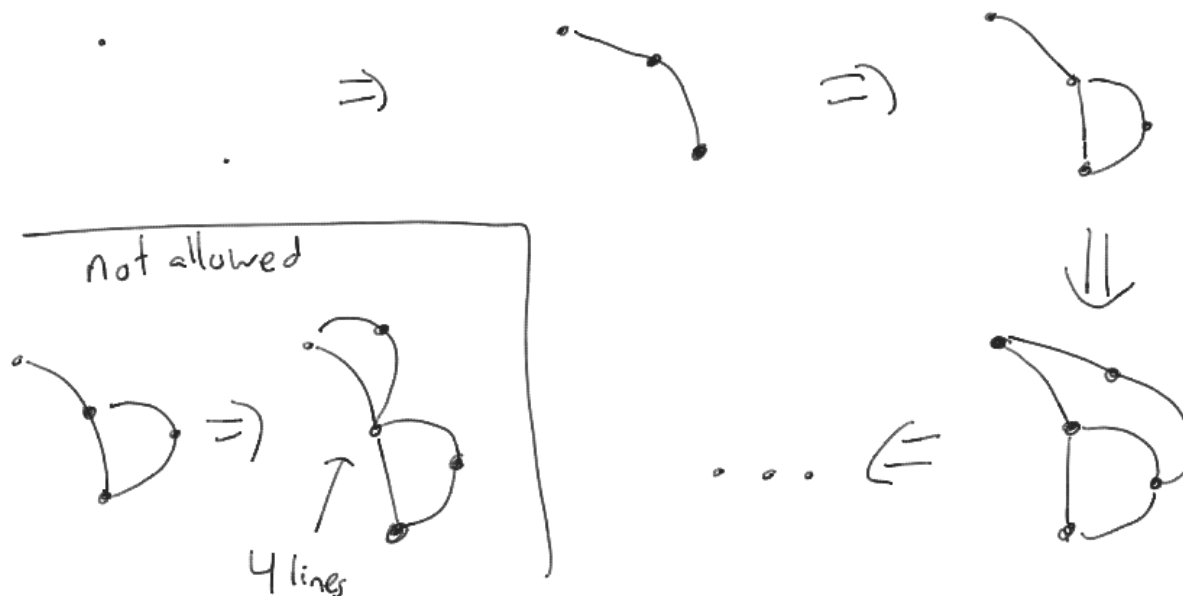
1.1. **The Game.** Sprouts is a fun two-player paper-and-pencil (or whiteboard-and-marker) game which has some interesting underlying mathematics.

**Setup:** Draw two dots (later you can explore with starting with more than two dots).

**Rules:** Players alternate taking turns. Each turn consists of drawing a line from one dot to another dot or to itself and then adding a new dot on that newly constructed line. When drawing new lines they must follow these rules:

- Lines can be curvy, but may not cross itself, another line, or pass through a dot.
- No dot can have more than three lines emanating from it (where a line starting and ending at the same dot counts as two).

**Objective:** The last player to make a legal move wins!



1.2. **Question.** After playing some games, you should think about the mathematics of sprouts. Here are a few questions to get you started.

- Does the first or second player always seem to win?
- Are there any tactics that you can use to help you win?
- How long can a game of sprouts go on for?
- Is there any pattern in the number of dots at the end of a game?
- Is there any pattern in the number of faces at the end of a game? (A face is a section of the paper bounded by lines. We also section of paper outside of any of the lines as a face.)
- What other patterns do you notice?
- What interesting variations can you make to the rules and setup?

## 2. BRUSSELS SPROUTS

2.1. **The Game.** Brussels Sprouts is a seemingly similar game to Sprouts.

**Setup:** Draw two crosses (later you can explore with starting with more than two crosses).

**Rules:** Players alternate taking turns. Each turn consists of extending an arm of a cross to another cross and then adding a dash to the newly constructed line to form a new cross. When drawing new lines they are allowed to be curvy, but may not cross itself, another line, or pass thru a cross.

**Objective:** The last player to make a legal move wins!

Play some games of Brussels Sprouts, and then ask and answer your own questions about Brussels Sprouts.

