Name: MAC2311

Period: **Discussion Quiz 8** November 4, 2021

1. On what intervals is the function $f(x) = x^3 + 3x^2 - 1$ increasing, decreasing, concave up, or concave down?

$f'(x) = 3x^2 + 6x = 3(x^2 + 2x) = 3x(x+2)$ $f'(x) = 0 \Rightarrow x = 0$ or $x = -2$

f': $+$ | -2 | $-$ | 0 | $+$

$f'(-3) = -9(-5) > 0$

$f'(-1) = -3(1) < 0$

$f'(1) = 3(3) > 0$

f is increasing on $(-\infty, -2)$ and $(0, \infty)$
decreasing on $(-2, 0)$

$f''(x) = 6x + 6$ $f''(x) = 0 \Rightarrow x = -1$

f'': $-$ | -1 | $+$

$f''(0) = 6 > 0$

$f''(-2) = -12 + 6 < 0$

f is concave down on $(-\infty, -1)$
concave up on $(-1, \infty)$

2. For each of the following functions, state whether or not the Mean Value Theorem can be applied on the interval $[-1, 1]$.

   If not, why not? [Don't write a whole paragraph, just one or two short sentences.]

   If yes, then there exists at least one point c in ***what interval*** such that the derivative of the function at $x = c$ is equal to ***what***? [Fill in the blanks with a specific interval and a specific number. You don't need to find c.]

   (a) $g(x) = \frac{1}{x+1}$

   MVT can not be applied because $g(x)$ is not continuous on $[-1, 1]$ (continuity fails at $x = -1$).

   (b) $h(x) = \frac{1}{x+2}$

   MVT can be applied.
   There exists at least one point c in $\boxed{(-1, 1)}$ such that the derivative of h at $x = c$ is equal to

$$\frac{h(1) - h(-1)}{1 - (-1)} = \frac{\frac{1}{1+2} - \frac{1}{-1+2}}{2} = \frac{\frac{1}{3} - 1}{2} = \frac{1}{6} - \frac{1}{2} = \frac{1}{6} - \frac{3}{6} = \boxed{-\frac{2}{6}}$$

Scoring for Discussion Quiz 8

Problem 1

Finding $f'(x)$: 0.5 points
Solving $f'(x) = 0$: 0.5 points
Finding where f' is positive/negative : 1 point
~~Correct~~
Finding $f''(x)$: 0.5 points
Solving $f''(x) = 0$: 0.5 points
Finding where f'' is positive/negative : 1 point

Stating the correct intervals : 1 point

Problem 2

a) Stating MVT cannot be applied : 1 point
~~Correct reasoning~~
Say this is because $g(x)$ is not continuous on $[-1, 1]$: 1 point

b) Stating MVT can be applied : 1 point
Stating c is in the interval $(-1, 1)$: 1 point
Stating $h'(c) = -\frac{2}{6}$: 1 point