

Name:

MAC2311

Period:

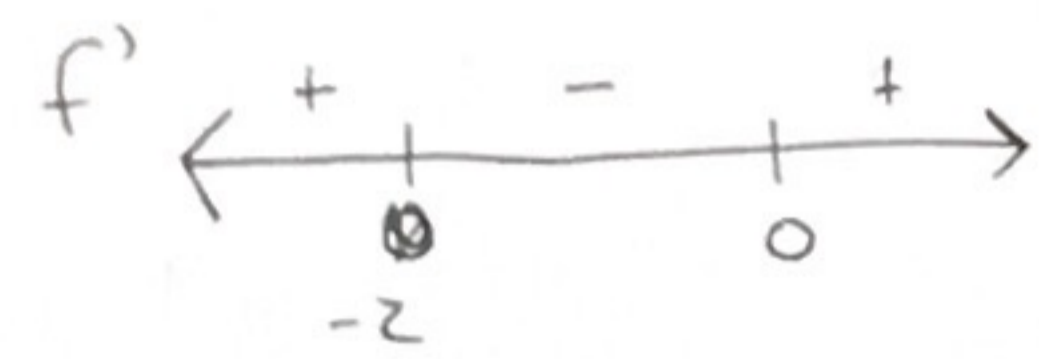
Discussion Quiz 8

November 4, 2021

1. On what intervals is the function  $f(x) = x^3 + 3x^2 - 1$  increasing, decreasing, concave up, or concave down?

$f'(x) = 3x^2 + 6x = 3(x^2 + 2x) = 3x(x+2)$

$f'(x) = 0 \Rightarrow x = 0$  or  $x = -2$



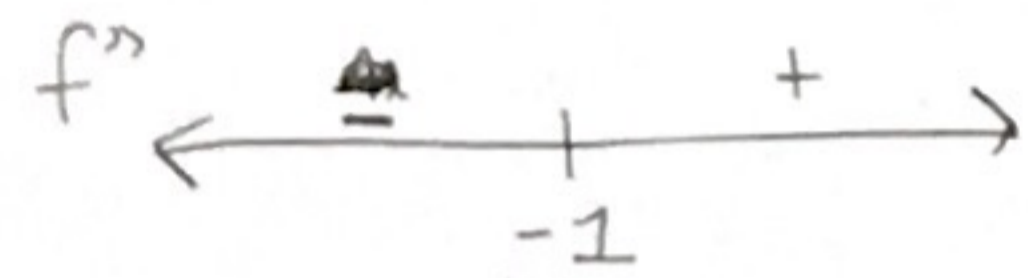
$f'(-3) = -9(-5) > 0$   
 $f'(-1) = -3(1) < 0$   
 $f'(1) = 3(3) > 0$

f is increasing on  $(-\infty, -2)$  and  $(0, \infty)$   
decreasing on  $(-2, 0)$   
f is concave down on  $(-\infty, -1)$   
concave up on  $(-1, \infty)$

$f''(x) = 6x + 6$   $f''(x) = 0 \Rightarrow x = -1$

$f''(0) = 6 > 0$

$f''(-2) = -12 + 6 < 0$



2. For each of the following functions, state whether or not the Mean Value Theorem can be applied on the interval  $[-1, 1]$ .

If not, why not? [Don't write a whole paragraph, just one or two short sentences.]

If yes, then there exists at least one point  $c$  in **what interval** such that the derivative of the function at  $x = c$  is equal to **what**? [Fill in the blanks with a specific interval and a specific number. You don't need to find  $c$ .]

(a)  $g(x) = \frac{1}{x+1}$

MVT can not be applied because  $g(x)$  is not continuous on  $[-1, 1]$  (continuity fails at  $x = -1$ ).

(b)  $h(x) = \frac{1}{x+2}$

MVT can be applied.

There exists at least one point  $c$  in  $(-1, 1)$

such that the derivative of  $h$  at  $x = c$  is

equal to  $\frac{h(1) - h(-1)}{1 - (-1)} = \frac{\frac{1}{1+2} - \frac{1}{-1+2}}{2} = \frac{\frac{1}{3} - 1}{2} = \frac{1}{6} - \frac{1}{2}$

$= \frac{1}{6} - \frac{3}{6} = \frac{-2}{6}$

Problem 1

Finding  $f'(x)$ : 0.5 points

Solving  $f'(x) = 0$ : 0.5 points

Finding where  $f'$  is positive/negative: 1 point

Finding  $f''(x)$ : 0.5 points

Solving  $f''(x) = 0$ : 0.5 points

Finding where  $f''$  is positive/negative: 1 point

Stating the correct intervals: 1 point

Problem 2

a) Stating MVT cannot be applied: 1 point

~~Correct reasoning~~  
Say this is because  $g(x)$  is not continuous on  $[-1, 1]$ : 1 point

b) Stating MVT can be applied: 1 point

Stating  $c$  is in the interval  $(-1, 1)$ : 1 point

Stating  $h'(c) = -\frac{2}{6}$ : 1 point