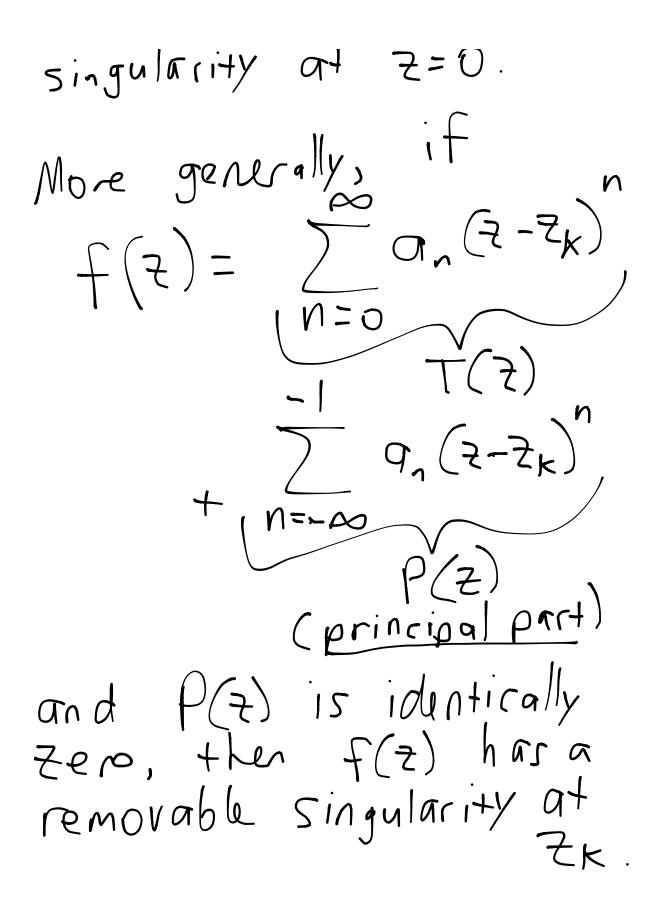
Residue Thm Let f be analytic on and interior to a simple closed curve &, except possibly at isolated singularities Z_1, Z_2, \ldots, Z_n $\int_{X} f(z) dz = 2\pi i \sum_{\substack{k=1 \ k=1}}^{n} \frac{\operatorname{Res} f}{z = z_{k}},$ where $\operatorname{Res} f$ is the notation $z = z_{k}$ for the residue of fat 7=ZK $f(z) = \sum_{n=0}^{\infty} O_n \left(z - \overline{z_k} \right),$

$$O \leq |Z-Z_{k}| \leq R_{1}$$

and the residue at $Z=Z_{k}$ is
precisely Q_{-1}
Isdated singularities come
in 3 flavors
D Removable singularities
 $eX = f(2) = \frac{1}{1!} + \frac{2}{2!} + \frac{2^{2}}{3!} + \frac{2}{3!} +$



2) Essential Singularity $\begin{array}{c|c} eX & | + \frac{1}{2} + \frac{1}{2!2^2} + \frac{1}{3!2^3} + \frac{1}{3!2^3} \\ & = e \\ & = e \\ & = e \\ & = 0 \\ & = 0 \\ \end{array}$ Here, the principal part has infinitely many terms. 3) Pole of finite order M, with JLM 200 $\frac{ex}{z} + \frac{1}{z} + \frac{z}{z} + \frac{z$

at Z=0. Since there is one term in the principal r part, et/2 has a pole of order <u>on</u> Order m of the pole = the largest degree of any term in the principal part. $\frac{E_X}{has} f(z) = \frac{1}{z^3} + |+z+\dots$ has a pole of order 3.

(2) Given f(z) with a pole of order m at $z=z_0$, how dowe calculate the residue? 2 approaches [] Use traditional power Series manipulations, much like the HW so far. This gives the full Laurent Series, and the residue is Q_1 21 Do the following.

$$f(z) = \sum_{N=0}^{\infty} q_{n} (z-z_{0})^{n} + \frac{q_{-1}}{z-z_{0}} + \dots + \frac{q_{-m}}{(z-z_{0})^{m}}$$

$$(\int (z-z_{0})^{m} f(z) = \sum_{N=0}^{\infty} q_{n} (z-z_{0})^{m-1} + \dots + q_{-m}$$

$$(\int \frac{d^{m-1}}{dz_{\infty}} (z-z_{0})^{m} f(z) \qquad n+1$$

The above calculation shows: The Let Zo be an isolated Singularity of f(z). Then Zo is a pole of order $m \ge 1$ of f if and only if there exists

 α function $\phi(z)$, holom. and nonzero in a NBAD of Zo, such that $f(z) - \phi(z)(z-z_0)$ Res $f(z) = \frac{\phi^{(m-1)}(z_0)}{(m-1)!}$ 2=70

