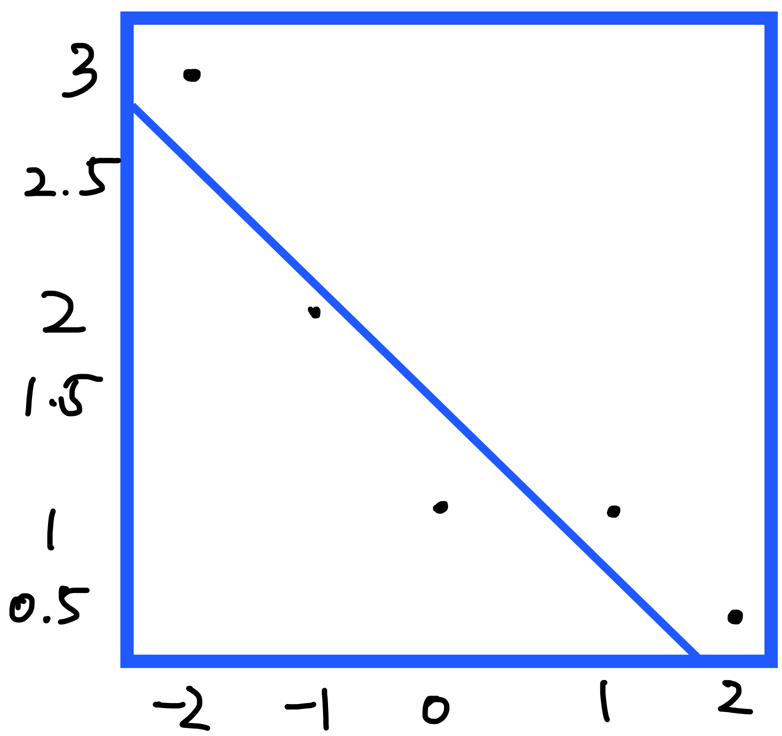
Formula: $S_{xy} = \sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})$ $S_{xx} = \sum_{i=1}^{n}(x_i-\bar{x})^2$ $S_{yy} = \sum_{i=1}^{n}(y_i-\bar{y})^2$

11.3. The summary statistics are $\bar{x}=0$, $\bar{y}=1.5$, $S_{xy}=-6$, $S_{xx}=10$. Thus $\hat{y} = 1.5 - 0.6x$



The graph looks like this

11.5. The summary statistics are $\bar{x}=4.5$, $\bar{y}=43.3625$, $S_{xy}=203.35$, $S_{xx}=42$. Thus

$$\hat{y} = 21.575 + 4.842x.$$

Since slope is positive, this suggests an increase in median prices over time. Also the expected annual increase is \$4842.

11.10. $\frac{dSSE}{d\beta_1} = -2\sum_{i=1}^{n}(y_i-\beta_1 x_i)x_i = -2\sum_{i=1}^{n}(x_i y_i - \beta_1 x_i^2) = 0$

$$\therefore \hat{\beta}_1 = \left(\sum_{i=1}^{n} x_i y_i\right) \Big/ \left(\sum_{i=1}^{n} x_i^2\right)$$

11.14 (a) The summary stats are

$\bar{x} = 0.325$ $\bar{y} = 0.755$ $S_{xy} = -0.27125$

$S_{xx} = 0.20625$

The line is $\hat{y} = 1.182 - 1.315x$

11.16 (a) The summary statistics are

$\bar{x} = 60$ $\bar{y} = 27$ $S_{xy} = -1900$ $S_{xx} = 6000$

The Least Squares Line is

$\hat{y} = 46.0 - 0.31667x$

(c) $SSE = S_{yy} - \hat{\beta}_1 S_{xy} = 792 - (-0.31667)\,1900$

$= 190.327$, $n = 12$,

$s^2 = SSE/(12-2) = 19.0327$

11.19 (a) $\bar{x} = 16$ $\bar{y} = 10.6$ $S_{xy} = 152$ $S_{xx} = 320$

Hence $\hat{y} = 3 + 4.75x$

(c) $s^2 = 5.025$

11.38. Refer to EX 11.3 and 11.23 where $s^2 = 0.1333$

$\hat{y} = 1.5 - 0.6x$, $S_{xx} = 10$ and $\bar{x} = 0$

When $x^* = 0$ the 90% C.I. for $E(Y)$ is $1.5 \pm 2.353 \cdot \sqrt{0.1333 \cdot \frac{1}{5}}$

$(1.12, 1.88)$

$\boxed{\text{Continued}}$

When $X^* = -2$, the 90% C.I. for $\mathbb{E}(Y)$ is $2.7 \pm 2.353\sqrt{0.1333\left(\frac{1}{5}+\frac{4}{10}\right)}$

$(2.03, 3.37)$

When $X^* = 2$, the 90% C.I. for $\mathbb{E}(Y)$ is

$0.3 \pm 2.353\sqrt{0.1333\left(\frac{1}{5}+\frac{4}{10}\right)}$ $(-0.37, 0.97)$

11.39. When $X^* = 65$, $\hat{y} = 25.395$. 95% C.I. for $\mathbb{E}(Y)$ is

$$25.395 \pm 2.228\sqrt{19.033\left[\frac{1}{12}+\frac{(65-60)^2}{6000}\right]} = 25.395 \pm 2.875$$

11.40. When $x^* = 0.3$ and $\hat{y} = 0.7878$ with $SSE = 0.0155$ and $S_{xx} = 0.20625$ and $\bar{x} = 0.325$. Then the 90% C.I. for $\mathbb{E}(Y)$ is

$$0.7878 \pm 1.86\sqrt{\frac{0.0155}{8}\left[\frac{1}{10}+\frac{(0.3-0.325)^2}{0.20625}\right]}$$

10.109 (a) Since for $X_i \sim \exp(\theta)$, $f_{X_i}(x) = \frac{1}{\theta}e^{-\frac{1}{\theta}x}$, $x>0$

Likelihood Function is

$$L(\theta) = \frac{1}{\theta_1^m \theta_2^n} e^{-\frac{1}{\theta_1}\sum_{i=1}^{m} X_i - \frac{1}{\theta_2}\sum_{j=1}^{n} Y_j}$$

Under $H_A: \theta_1 \neq \theta_2$, MLE of them are $\hat{\theta}_{1MLE} = \bar{X}$, $\hat{\theta}_{2MLE} = \bar{Y}$

Under: $H_0: \theta_1 = \theta_2$, MLE of θ is $\hat{\theta}_{MLE} = \dfrac{\sum_i X_i + \sum_j Y_j}{m+n}$

Hence $\Lambda = \dfrac{\sup_{\theta \in \Theta_0} L(\theta)}{\sup_{\theta \in \mathbb{R}^2} L(\theta)}$

Continued

In the nominator

$$\log \frac{1}{\theta^{n+m}} e^{-\frac{1}{\theta}(\sum_{i=1}^{m} x_i + \sum_{j=1}^{n} y_j)} = -(n+m)\log\theta - \frac{1}{\theta}\left(\sum_i x_i + \sum_j y_j\right)$$

$$\frac{\partial}{\partial\theta}\log L = -\frac{n+m}{\theta} + \frac{1}{\theta^2}\left(\sum_i x_i + \sum_j y_j\right)$$, Set it as 0

Hence $\hat{\theta}_{MLE} = \frac{1}{n+m}\left(\sum_i x_i + \sum_j y_j\right)$

Similarly in the denominator

$$\log L = -m\log\theta_1 - n\log\theta_2 - \frac{1}{\theta_1}\sum_i x_i - \frac{1}{\theta_2}\sum_j y_j$$

Set $\frac{\partial \log L}{\partial\theta_1}$ and $\frac{\partial \log L}{\partial\theta_2} = 0$ separately, we can know

$\hat{\theta}_{1MLE} = \bar{X}$ and $\hat{\theta}_{2MLE} = \bar{Y}$

Then we have that

$$\Lambda(\underline{x}, \underline{y}) = \frac{\left(\frac{n+m}{\sum_i x_i + \sum_j y_j}\right)^{n+m} e^{-\frac{n+m}{\sum_i x_i + \sum_j y_j}\left(\sum_i x_i + \sum_j y_j\right)}}{\left(\frac{m}{\sum_i x_i}\right)^m \left(\frac{n}{\sum_j y_j}\right)^n e^{-\frac{m}{\sum_i x_i}\sum_i x_i} e^{-\frac{n}{\sum_j y_j}\sum_j y_j}}$$

$$= \frac{(n+m)^{n+m}}{n^n m^m} \frac{\left(\sum_i x_i\right)^m \left(\sum_j y_j\right)^n}{\left(\sum_i x_i + \sum_j y_j\right)^{n+m}}$$