

STA 4321/5325

Spring 2010

Exam 2

Full Name: \_\_\_\_\_

*On my honor, I have neither given nor received unauthorized aid on this examination.*

Signature: \_\_\_\_\_

This is a 50 minute exam. There are 4 problems, worth a total of 40 points. **Remember to show your work.** Answers lacking adequate justification may not receive full credit. You may use one letter-sized sheet (the same size as the lecture notes) of your own notes and a pocket calculator. (You are not required to bring a calculator — you may leave your answers in a form from which the numerical answer could be immediately calculated.) You may *not* use any books, other references, or text-capable electronic devices.

1. Let  $X$  be the number of heads observed in 4196 tosses of a fair coin.
  - (a) Which subcollection of discrete random variables does  $X$  belong to? [2 pts]
  - (b) Provide  $E(X^2)$ . You may use identities established in class about this specific subcollection of discrete random variables.  
Hint: Write down  $E(X^2)$  in terms of  $E(X)$  and  $V(X)$ . [4 pts]
  - (c) Find the probability that the observed number of heads is equal to the observed number of tails. [4 pts]
  
2. A car saleswoman has to sell 1 car. She is provided with a very large (infinite for practical purposes) list of customers. She approaches customers sequentially according to the list. The probability that she makes a successful sale to any given customer is 0.2. She stops as soon as she sells the car. Suppose that all the customers behave independently of each other. Let
$$X = \text{Number of failed sales before a successful sale.}$$
  - (a) Which subcollection of discrete random variables does  $X$  belong to? [2 pts]
  - (b) Given that the first 5 sales were failed sales, find the probability that there will be atleast 9 more failed sales before a successful sale. [4 pts]
  - (c) Find the expected value and variance of the **total** number of customers the saleswoman has to approach. [4 pts]
  
3. Customer arrivals at a checkout counter in a department store have a Poisson distribution with an average of forty nine per hour.
  - (a) Find the probability that exactly 25 customers arrive in a given hour. [2 pts]
  - (b) Find the probability that atleast one customer arrives in a given hour. [4 pts]
  - (c) Provide  $P(X = 0 \mid X \leq 1)$ . [4 pts]

4. A company has 30 personal computers (PCs) in its warehouse. Although all are new and still in boxes, four do not currently function properly. One of the company's offices requests 18 PCs, and the warehouse foreman selects 18 from the stock of 30 and ships them to the requested office. Let

$X =$  Number of PCs that are defective.

- (a) Which subcollection of discrete random variables does  $X$  belong to? [2 pts]
- (b) Find the probability that all 18 of the PCs are not defective. [4 pts]
- (c) Find  $E[(X + 4)^2] - V(X)$ . You may use identities established in class about this specific subcollection of discrete random variables.

Hint: Use  $(t + 4)^2 = t^2 + 4t + 16$  followed by linearity of expectations. [4 pts]