Quiz 3 (with solution hints)

Full Name: _____

On my honor, I have neither given nor received unauthorized aid on this quiz

Signature: _____

This is a 10 minute quiz. There are 5 multiple choice problems, each having EXACTLY ONE correct answer. You may *not* use any books, other references, or text-capable electronic devices.

1. Let X be the number of heads obtained in 100 independent tosses of a fair coin. Then the Range of X is given by

    (a) $\{0, 2, 4, \cdots, 98, 100\}$.
    (b) $\{1, 3, 5, \cdots, 97, 99\}$.
    (c) $\{0, 1, 2, \cdots, 99, 100\}$.
    (d) All non-negative integers.

    Correct answer (c) (see lecture notes).

2. A six-faced die is cast, and let X denote the number on the face that shows up. Assuming that all faces have the same chance of showing up, the expected value of X is given by

    (a) $\frac{21}{6}$.
    (b) $\frac{7}{2}$.
    (c) $\frac{21}{2}$.
    (d) $\frac{15}{6}$.

    Correct answer (a) & (b) (this was a typo on my part, and I gave full points for either of the two choices). The range of X is all integers between 1 and 6, and each value is taken with equal probability $\frac{1}{6}$. Hence,

    $$E[X] = \frac{1}{6}(1 + 2 + 3 + 4 + 5 + 6) = \frac{21}{6} = \frac{7}{2}.$$

3. Suppose $E[X] = 5$ and $E\left[X^2\right] = 32$. Then

    (a) $V(X) = 27$.
    (b) $V(X) = 37$.
    (c) $V(X) = 25$.
    (d) $V(X) = 7$.

    Correct answer (d): $V(X) = E[X^2] - (E[X])^2 = 32 - 25 = 7$.

4. If $E[X] = 22$ and $E[Y] = 8$, then

   (a) $E[X + Y] = 30$.
   (b) $E[X + Y] = 548$.
   (c) $E[X + Y] = 14$.
   (d) $E[X + Y] = 27$.

   Correct answer (a): By linearity of expectations, $E[X + Y] = E[X] + E[Y] = 30$.

5. A Bernoulli random variable takes only two possible values, 0 and 1. This statement is

   (a) True.
   (b) False.

   Correct answer (a) (see lecture notes).