SOLUTIONS TO EXAM 3

PROBLEM 1: SEE SOLUTIONS TO PROBLEM 4 OF THE SAMPLE EXAM

PROBLEM 2: [a] Note that the rejection region of the standard level-0.05 test for this problem is given by

$$\text{REJECT } H_0 \text{ if } \quad \bar{Y} > 18 + z_{0.95} \times \frac{S}{\sqrt{100}}$$

Based on the formula provided in class,

$$\beta(20) = \Phi\left(\frac{18-20}{5/\sqrt{100}} + z_{0.95}\right)$$

$$= \Phi\left(-4 + 1.68\right)$$

$$= \Phi(-2.32)$$

[b] Based on the formula provided in class, the minimum sample size to ensure $\beta(20) = 0.01$ is given by

$$n = \frac{(z_{0.95} + z_{0.99})^2 \times 25}{(20-18)^2} = \frac{(1.68+2.3)^2 \times 25}{4}$$

$$= 99.025$$

Problem 3: (a) Let μ denote the average hourly wage paid by the company. The above problem can be formulated as testing the hypotheses

$$H_0: \mu = 13.20 \quad \text{vs.} \quad H_A: \mu < 13.20$$

based on a random sample of 40 workers from the company

(b) Since μ is the population average, an appropriate test statistic is the sample mean

$$\bar{Y} = \frac{1}{400} \sum_{i=1}^{400} Y_i$$

(c) Based on the large sample testing procedure derived in class, the rejection region of the level-0.05 test based on $\bar{Y}$ is given by

$$\text{Reject } H_0 \text{ if } \bar{Y} < 13.20 + z_{0.05} \frac{5}{\sqrt{40}}$$

$$\text{OBSERVED VALUE} \rightarrow \boxed{12.20} < 13.20 - \frac{1.68 \times 2.50}{\sqrt{40}} \approx \boxed{12.54}$$

Hence, based on the observed data, there is enough evidence to suggest that the company is paying substandard wages.

Problem 4: (a) We will reject the null hypothesis for large values of $\hat{\theta}$.

(b) Based on the formula provided in class, the p-value is given by

$$P(\hat{\theta} \geq \hat{\theta}_{observed} \mid H_0 \text{ is true})$$

$$= P\left(\frac{\hat{\theta} - 15}{\hat{SE}(\hat{\theta})} \geq \frac{19 - 15}{\hat{SE}(\hat{\theta})} \mid \theta = 15\right)$$

$$= P\left(\text{Normal}(0,1) \geq \frac{4}{5} \mid \theta = 15\right)$$

$$= 1 - \Phi(0.8)$$

↳ CDF of Normal (0, 1)

(c) Since $1 - \Phi(0.8) > 0.05$, we will accept H_0 at level 0.05.